Transcomputation

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Agenda - size matters!

- Total set of relational operators: less than, equal to, greater than

- Sketching transreal graphs

Relational operators

Quadrachotomy

- Transreal arithmetic is defined by axioms
- Real arithmetic obeys the axiom of trichotomy: every number falls into exactly one of three cases: less than zero, equal to zero, greater than zero
- Transreal arithmetic allows a fourth, distinct case: equal to nullity

No undefined results

- In real arithmetic any result that reduces to $0/0$ is said to be undefined
- How do we know there are no undefined results in transreal arithmetic?

Greater than

- Define: $x > y \Leftrightarrow x - y > 0$

- How does real arithmetic determine if a number is greater than zero?

- How does transreal arithmetic determine if a number is greater than zero?

Infinity is big!

Theorem: infinity is bigger than any real number

Axiom: $\infty > 0$

Proof: let r be an arbitrary real number

Then $\infty - r = \frac{1}{0} - \frac{r}{1} = \frac{1 \times 1 - 0 \times r}{0 \times 1} = \frac{1}{0} = \infty > 0$

Infinity compared to itself

Theorem: infinity is not bigger or smaller than itself

Proof: $\infty - \infty = \Phi \neq 0$

Infinity compared to itself

- How do we know that infinity is not equal to nullity?
- How do we know that infinity is equal to itself?

No indefinite results

- In real arithmetic there are two kinds of indefinite results that do not have a precise numerical value
- Any result that reduces to 1/0 is indefinitely large
- Any result that reduces to -1/0 is indefinitely small
- How do we know there are no indefinite results in transreal arithmetic?

Nullity compared to itself

- How do we know that nullity is not less than, equal to, or greater than any other number?
- How do we know that nullity is equal to itself?

Relational operators

- The three relational operators less than, equal to, greater than are total
- In the next three slides r is a real number and * means that the case has to be evaluated to determine its truth value

Less than

<	-∞	r2	∞	Φ
-∞	F	T	T	F
r1	F	*	T	F
∞	F	F	F	F
Φ	F	F	F	F

Equal to

=	-∞	r2	∞	Φ
-∞	T	F	F	F
r1	F	*	F	F
∞	F	F	T	F
Φ	F	F	F	T

Greater than

>	-∞	r2	∞	Φ
-∞	F	F	F	F
r1	T	*	F	F
∞	T	T	F	F
Φ	F	F	F	F

Relational operators

- Three primitive, relational operators: <, =, >
- Can be combined into $2^3 = 8$ compound operators
- Seven of the operators are: <, =, >, <=, >=, <>, <=>,
- Which 8th operator is missing from this list?

Relational operators

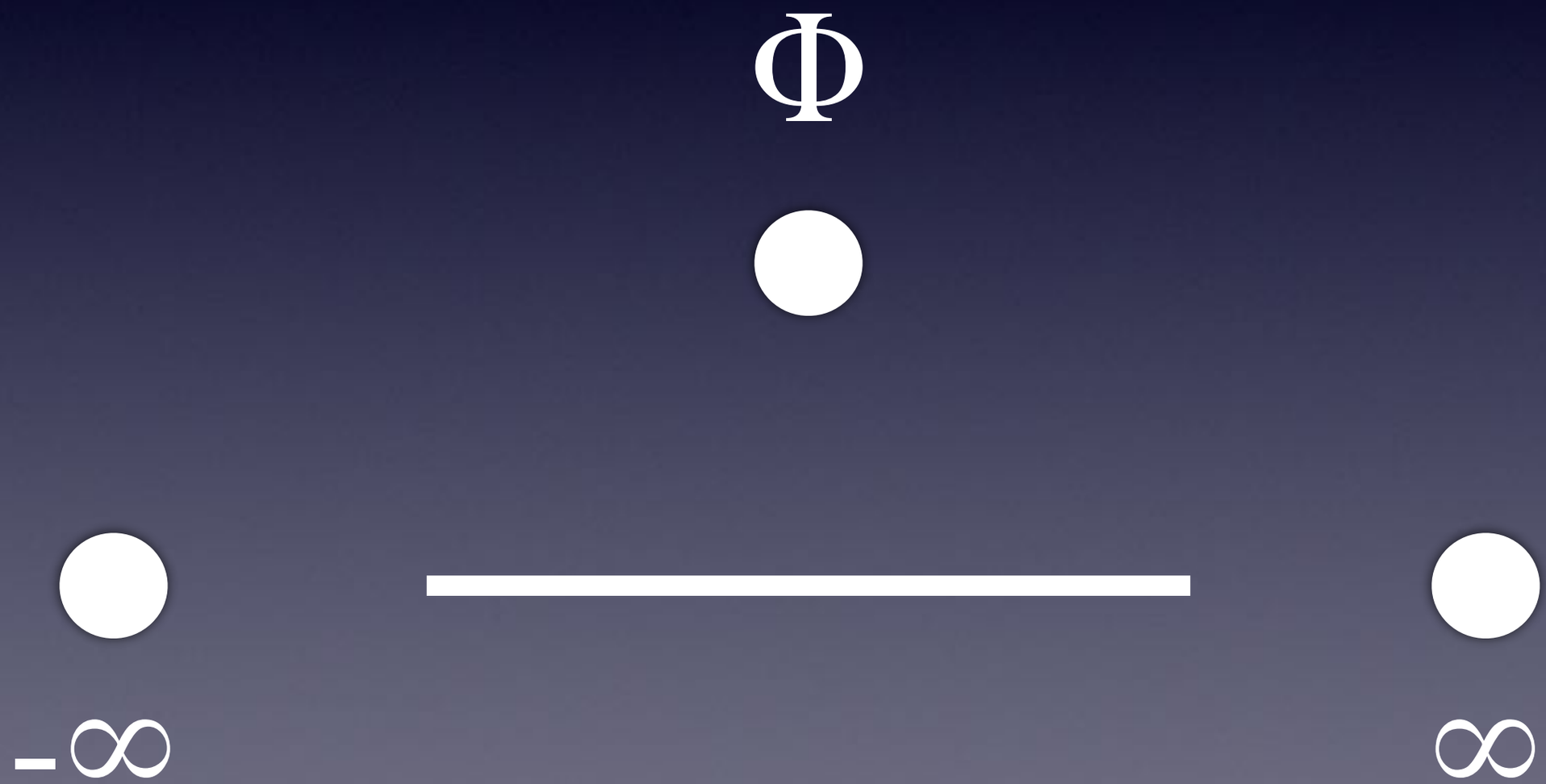
- The 8 relational operators can be combined with negation (!) to give 16 operators
- What is the negation of the 8th operator that was missing from the above list?

Relational operators

- Rationale for total set of relational operators
- Total set of relational operators

Sketching

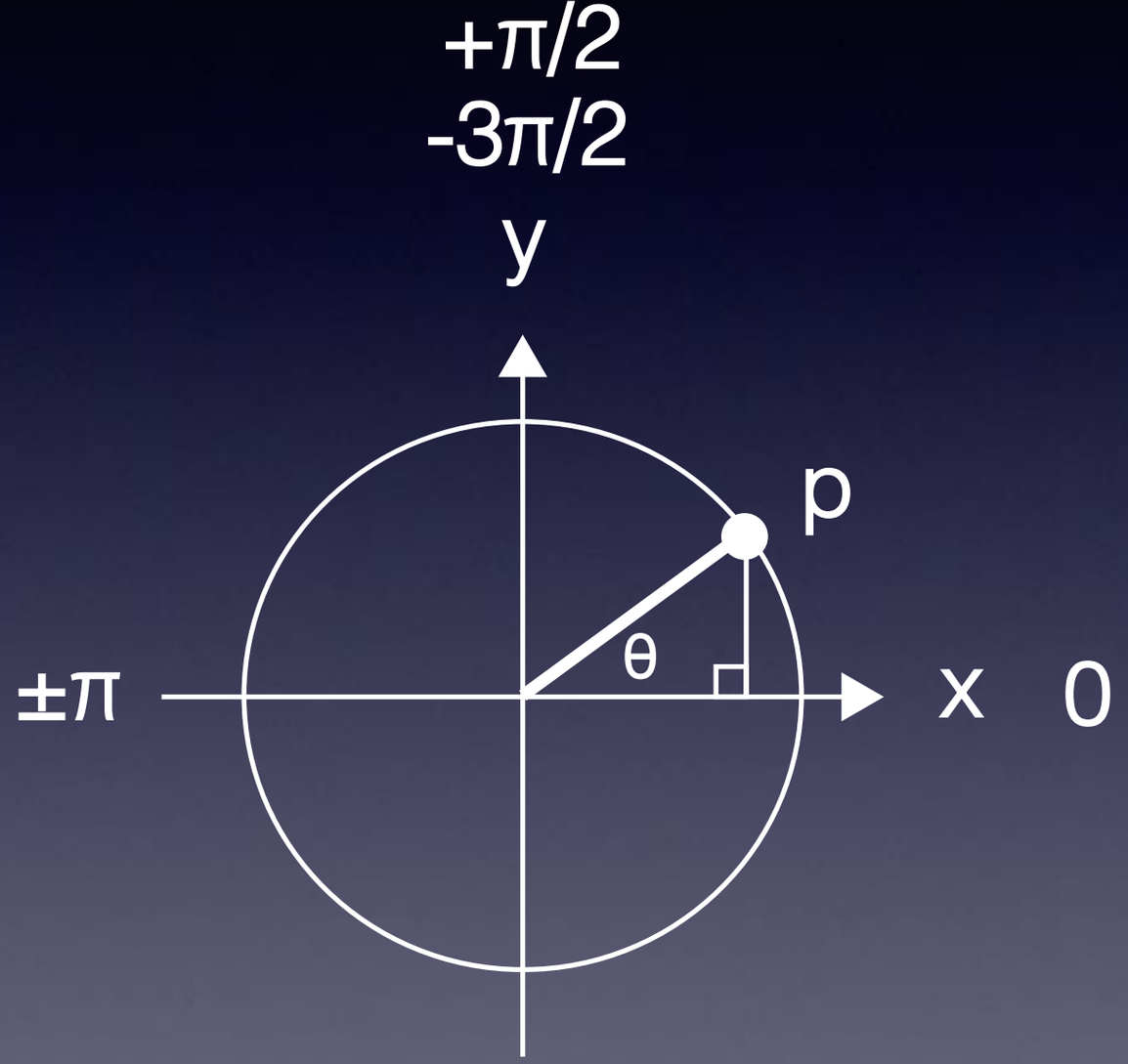
Transreal-Number Line



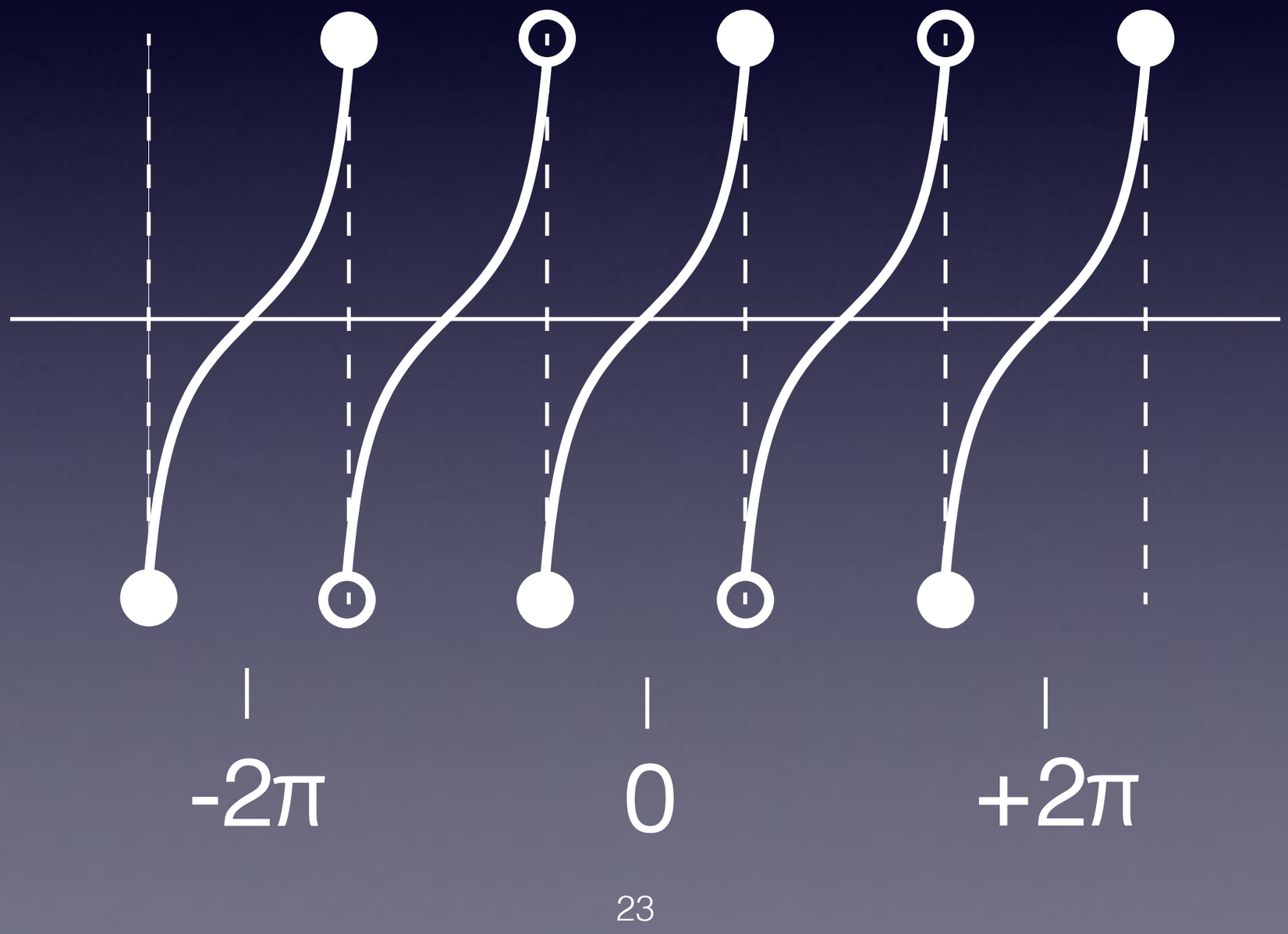
Graphs

- Each axis is a transreal number line
- Put nullity in a convenient or aesthetically pleasing place
- Use the usual conventions for sketching graphs

Tangent



Tangent



Totallity

- Which angles are missing from the above sketched graph of the tangent?

- What is the value of the tangent at these angles?

Conclusion

- There are no indefinite or undefined results in transreal arithmetic
- Transreal, relational operators are total
- We can sketch transreal functions