Transcomputation

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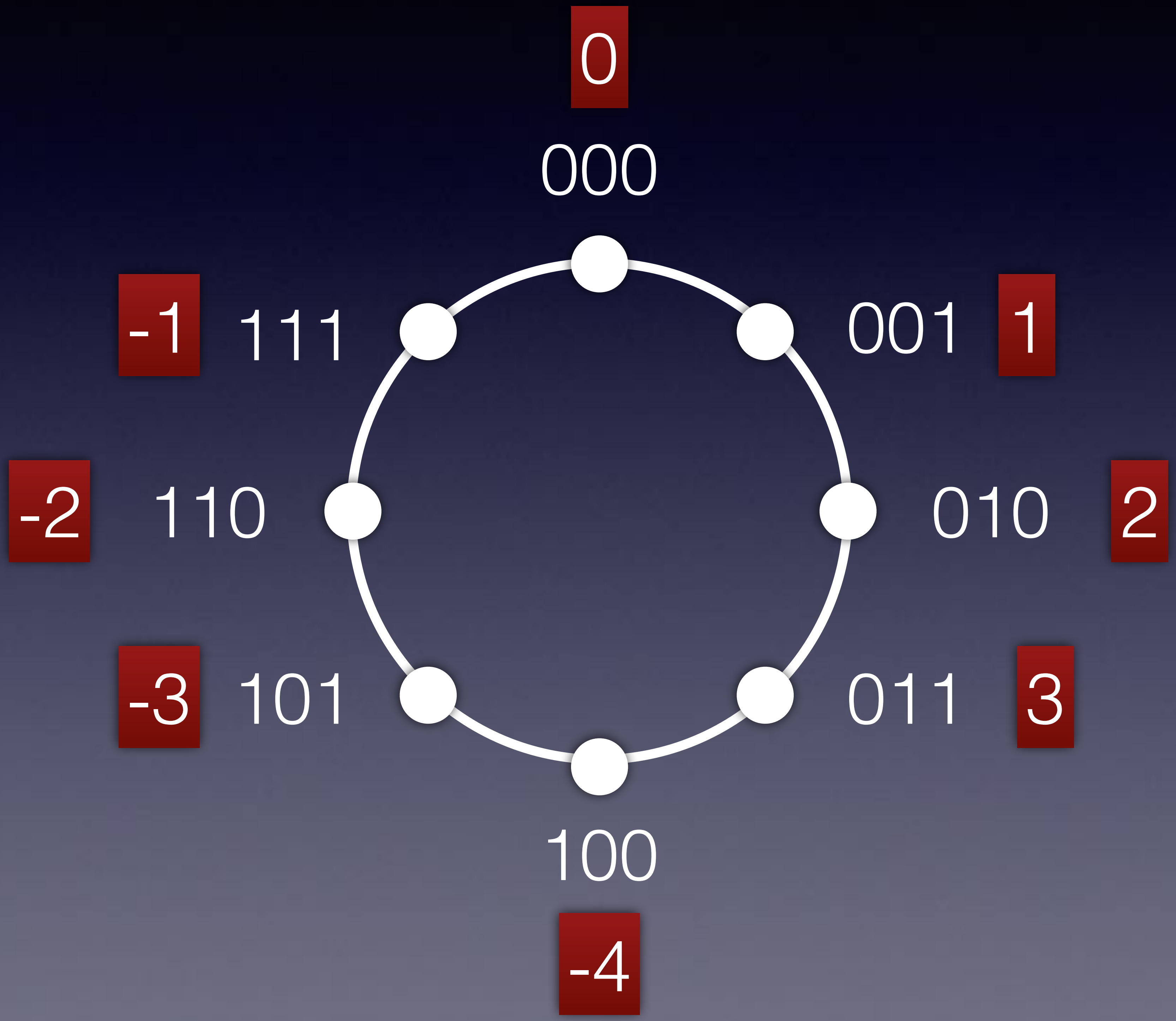
Agenda

- Trans-two's-complement numbers
- Transfloating-point numbers

Trans-two's complement

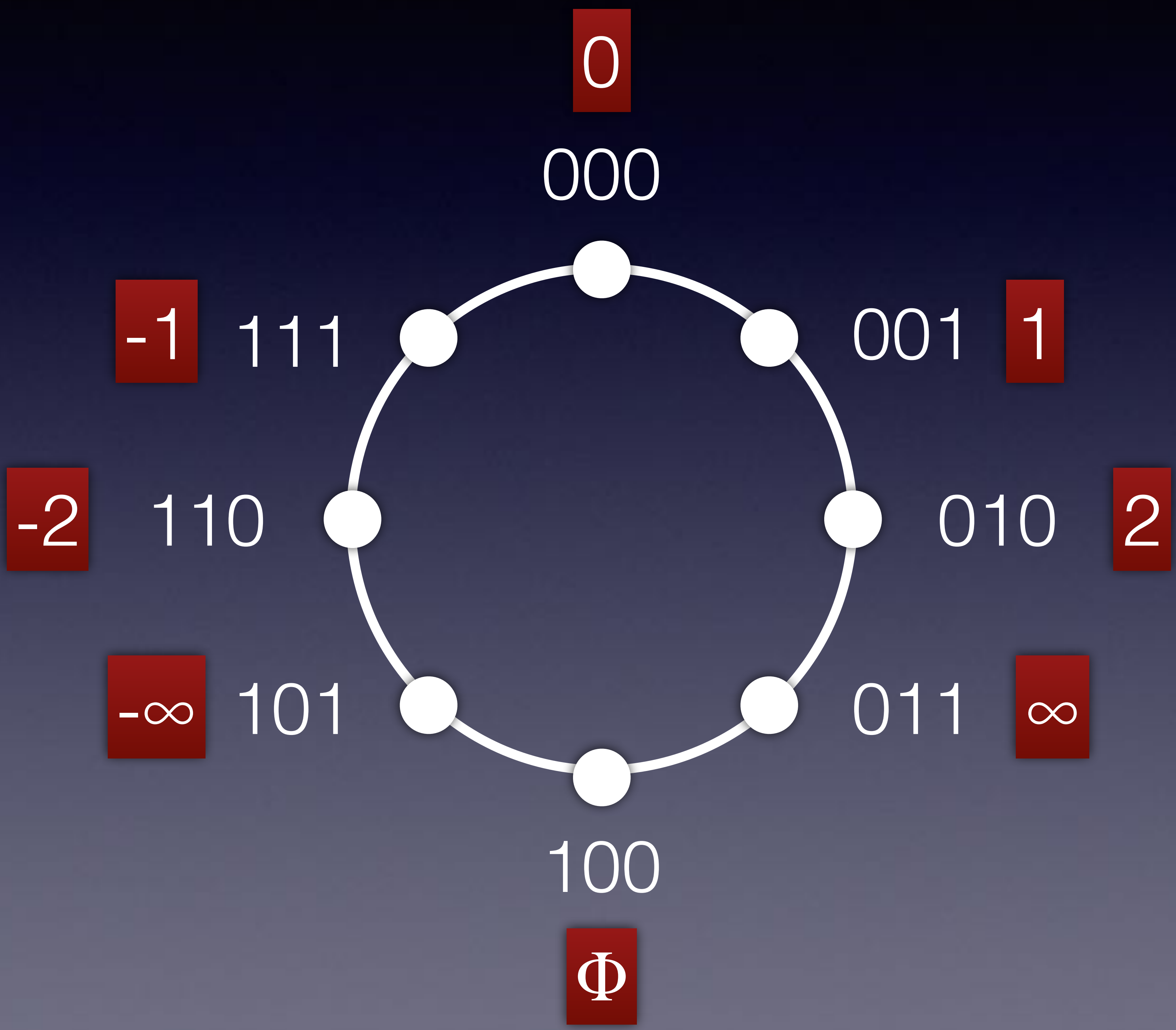
Two's complement



Two's complement

- One more negative than positive number
- Wrap-around error
- Weird-number error
- Prioritises range over correctness!
- Different topology from floating-point
- Different topology from real numbers

Trans-two's complement



Trans-two's complement

- Same number of positive and negative numbers
- No wrap-around error
- No weird-number error
- Obtains maximal range but with round-off to infinities
- Same topology as transfloats
- Discrete approximation to topology of real numbers

IEEE 754
Floating-point

Float zeros

X 00000000000 00000000000000000000

00000000000000000000000000000000

X = 0 → +0

X = 1 → -0

Float infinities

X11111111111 00000000000000000000

00000000000000000000000000000000

X $= 0 \rightarrow +\infty$

X $= 1 \rightarrow -\infty$

Float NaNs

X 11111111111 XXXXXXXXXXXXXXXXXXXX

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Ignore X and at leat one X = 1

So 9,007,199,254,740,990 NaN states

Float NaNs

- Unequal to themselves so break the semantics of equality

Quiz

- At 64-bits, how many NaNs are there?

Transfloating-point

Transfloat zero & nullity

X 00000000000 00000000000000000000

00000000000000000000000000000000

X $= 0 \rightarrow 0$

X $= 1 \rightarrow \Phi$

Transfloat infinities

X11111111111 11111111111111111111

11111111111111111111111111111111

X = 0 → $+\infty$

X = 1 → $-\infty$

Quiz

- At 64-bits, how many more transfoating-point numbers are there than IEEE 754 floating-point numbers?

Abolishing NaNs

- Doubles real range - giving larger numbers
- Or halves size of smallest, positive number - potentially doubling accuracy

Abolishing minus zero

- Preserves mathematical semantics of zero
- Preserves transmathematical semantics of zero

Transfloat

- No NaNs
- Twice the range of real numbers mapped to a potential doubling of accuracy, by halving the smallest, representable number
- Preserves the semantics of zero and equality
- Same topology as trans-two's complement

Independent relational operators

Floating-Point

- Four primitive, relational operators: <, =, >, ?
- Said to be mutually independent
- So must be $2^4 = 16$ compound operators
- But there are only 11 compound operators so the standard is wrong!
- With negation, wastes 10 states

Mathematics

- Three primitive, relational operators: <, =, >
- Can be combined with not: !
- But all of the negations, except not-epsilon, are redundant so wastes 7 states

Trans float/maths

- Three primitive, relational operators: <, =, >
- Can be combined with not: !
- Giving $2^4 = 16$ compound operators
- Relational operators AND negation are independent so no wasted states

Relops and Negation

- IEEE 754 floats waste 10 states
- Mathematics wastes 7 states
- Trans float/maths waste 0 states

Total Order

- Trans float/maths is totally ordered in the extended-real numbers, with nullity as the uniquely unordered, transreal number

- IEEE 754 relational operator, TotalOrder, has the category error that a total ordering of bit patterns produces a total unorder of floating-point objects because -NaNi < F < NaNi for all NaNs and all floating-point numbers F.

Conclusion

- Trans-two's complement has the same number of positive and negative numbers and removes the weird number and wrap-around
- Has the same topology as the transfloats
- Is a discrete approximation to the topology of the transreals

Conclusion

- Transfloat preserves the semantics of zero and equality, potentially doubles the accuracy of float calculations, has irredundant relops, is totally ordered when a position is imposed on nullity
- Has the same topology as trans-two's-complement
- Is a discrete approximation to the topology of the transreals