Transcomputation - Exercise 6

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Note

In this Exercise polar-transcomplex numbers are written in parentheses as trans-tuples of the form (r, θ), where r and θ are transreal numbers, and Cartesian transcomplex-numbers are written in square brackets as trans-tuples of the form $[x, y]$, where x and y are transreal numbers.

1 Transcomplex sums

1.1 Every polar trans-tuple (r, θ) can be written uniquely as a Cartesian trans-tuple $[r \cos \theta, r \sin \theta]$. How do you know there is (a) at least one and (b) no more than one Cartesian trans-tuple for each polar trans-tuple? In other words, how do you know that each polar trans-tuple corresponds to exactly one Cartesian trans-tuple?

1.2 Give an example of two different Cartesian trans-tuples that correspond to the same polar trans-tuple.

1.3 Convert these two finite polar tuples $a = (2, 0)$, $b = (2, \pi/4)$, to the corresponding Cartesian trans-tuples a' and b'.

1.4 Compute the Cartesian sum $c' = a' + b'$.

1.5 Convert the Cartesian complex number c' to polar form c. Now c is the polar sum $c = a + b$.

1.6 Compute the sum $(\infty, 0.5) + (\infty, 0.6)$.

1.7 Compute the sum $(\infty, 0.5) + (\infty, -0.5)$.

1.8 Compute the sum $(\Phi, 3) + (\infty, 6)$.

1.9 Compute the sum $(2, \infty) + (3, 4)$.

2 Transcomplex division

2.1 Prove that the division formula, $(r_1, \theta_1) \div (r_2, \theta_2) = (r_1/r_2, \theta_1 - \theta_2)$, calculates infinity correctly if and only if the angle of zero is zero.

2.2 Given that the angle of zero is zero, use the division formula to calculate the angle of nullity.