Perspex Machine Tutorial

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Introduction

- So you think you appreciate the homunculus problem?
- Trans numbers.
- Robust trigonometry.
- The perspex machine.
- Perspex neural nets.
- Conclusion.

Homunculus

- It was once thought that there is a fully formed little man or woman, called a homunculus, inside each spermatozoon and egg. This is a mistake. No homunculus is needed to explain sexual reproduction.

- It was once thought that there is a homunculus in the pineal body in the brain that lies at the focus of the two eyes. This homunculus was supposed to do the seeing for the brain it inhabited. This is a mistake. Stereo vision can be explained without a homunculus.

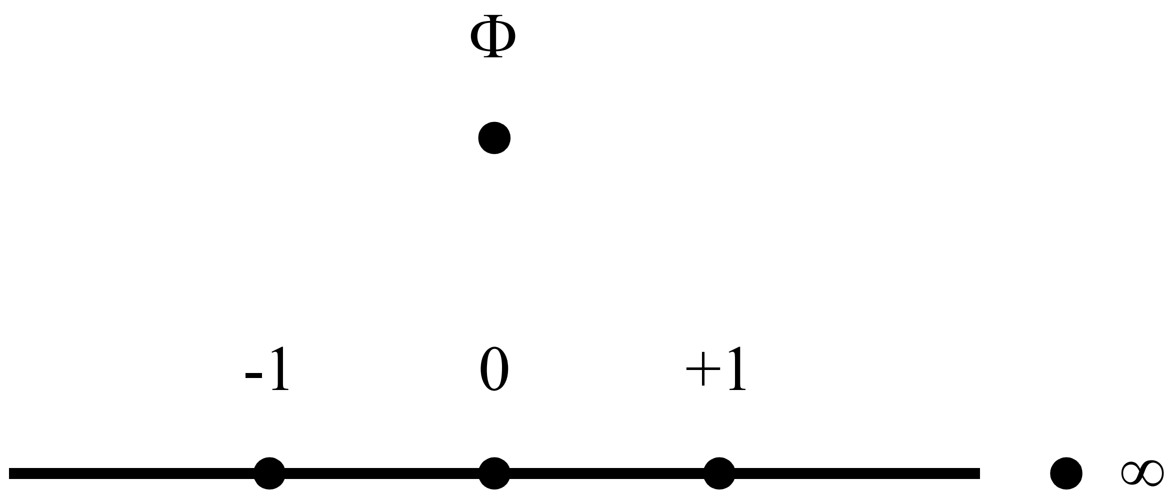
- It was once thought that division by zero cannot occur in arithmetic so that a human mind, or homunculus, is needed to resolve paradoxes. This too is a mistake, but a more recent one!

Trans Numbers

- $S = \left\{ \frac{n}{d} : n, d \in Z; \left(d^2 - n^2\right)^2 + \left(2dn\right)^2 = \left(d^2 + n^2\right)^2 \right\}.$
- Then, choosing a congruence that establishes rational trigonometry, $S \equiv Q \cup \left\{ \frac{1}{0}, \frac{0}{0} \right\}.$
- Hence infinity, $\frac{x}{0} \equiv \frac{1}{0} = \infty$ for all real $x \neq 0$, and nullity, $\frac{0}{0} = \Phi$, are numbers because they are fractions that occur in S.

Trans Numbers

- The number infinity lies at the positive extreme of the number line.
- The number nullity lies off the number line.
- The number nullity means that there is no unique number on the number line that satisfies a given equation.



Questions

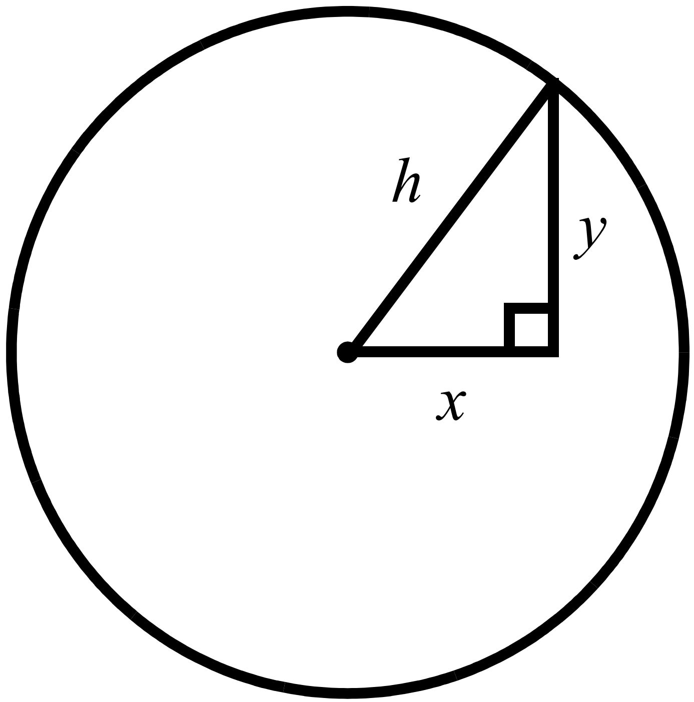
- At what time in seconds, taking *now* as zero, do you stop beating your wife or husband?
- What is the known date of birth of a foetus in utero?
- In a projective geometry of dimension at least three, what are the co-ordinates of the point at which skew lines meet?
- The Jacobi algorithm for liberating eigen systems from a square, symmetric matrix risks a division by zero. Why does allowing the division by zero produce a correct algorithm that is simpler and faster?
- Is the invention of nullity as significant as that of zero?

Trans Numbers

- Transintegers are $Z \cup \{\Phi, \infty\}$. Transinteger arithmetic contains integer arithmetic and is consistent with it. Similarly for:
- Transrational arithmetic;
- Transreal arithmetic;
- Transcomplex arithmetic;
- Transquaternion arithmetic.
- (I have not yet examined octonion arithmetic in enough detail to know if it is consistent with transoctonion arithmetic.)

Robust Trigonometry

- Suppose we want to perform rational trigonometry where the sides x, y, and $h = 1$ of a right angled triangle, as shown, are rational numbers.



Robust Trigonometry

- Transrational arithmetic provides a total, rational trigonometry.
- Transrational trigonometry is exact within its own number system.
- These two properties are helpful in practical trigonometric computations, say in computer vision, because:
- A solution is always available;
- The trigonometric calculations are infinitely robust regardless of the sensitivity of the problem.

Perspex Machine

- The perspex machine operates in a 4D space called “perspex space.”
- The points in perspex space have transreal co-ordinates.
- Every point in perspex space contains a 4×4 matrix of transreal co-ordinates.
- The perspex machine is started at a point or points in perspex space.
- The perspex machine may be started at every point on a segment of the real number line. This makes the perspex machine super-Turing.

Perspex Matrix & Transformation

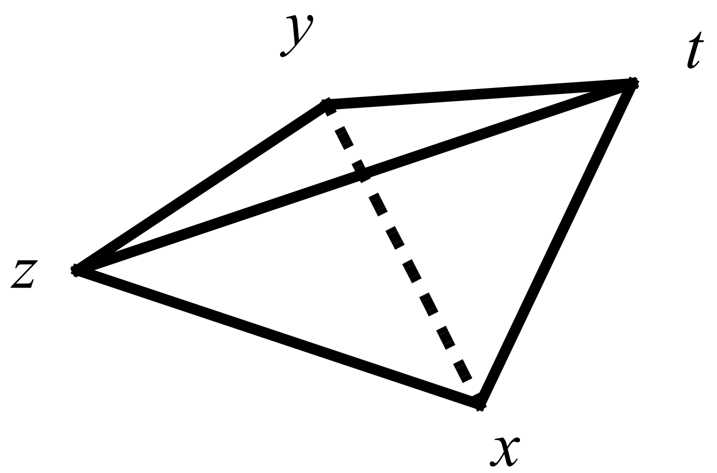
- The perspex is conventionally laid out as four column vectors x, y, z, t:

$$\begin{bmatrix} x_1 & y_1 & z_1 & t_1 \\ x_2 & y_2 & z_2 & t_2 \\ x_3 & y_3 & z_3 & t_3 \\ x_4 & y_4 & z_4 & t_4 \end{bmatrix}.$$

- Thus, the perspex can describe geometrical transformations of the world.

Perspex Tetrahedron

- The column vectors of a perspex can describe the vertices of a tetrahedron.



- Thus, the perspex can describe the geometrical structure of the world.

Perspex Instruction

- A perspex at point p can be the instruction $(\overrightarrow{x+p})(\overrightarrow{y+p}) + \text{continuum}(\overrightarrow{z+p}) \rightarrow \overrightarrow{z+p};$ $\text{jump}(\overrightarrow{z_{11}+p}, t)$.
- The superscript arrow denotes indirection. For example, x is a column vector denoting a position in 4D space, but $\vec{x}$ is the contents of that point.
- Matrix multiplication and addition are shown as usual.
- Transreal arithmetic make the perspex machine super-Turing.

Halting Instruction

- $H = \begin{bmatrix} \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi \\ \Phi & \Phi & \Phi & \Phi \end{bmatrix}.$
- Executing *H* halts the perspex machine.
- The perspex machine is programmed by initialising space with, typically, non-*H* perspexes.

Projection on to the Continuum

- $N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$

- $\text{continuum}(a) = \begin{cases} a, & a \neq H \\ N, & a = H \end{cases}.$

Control Jump

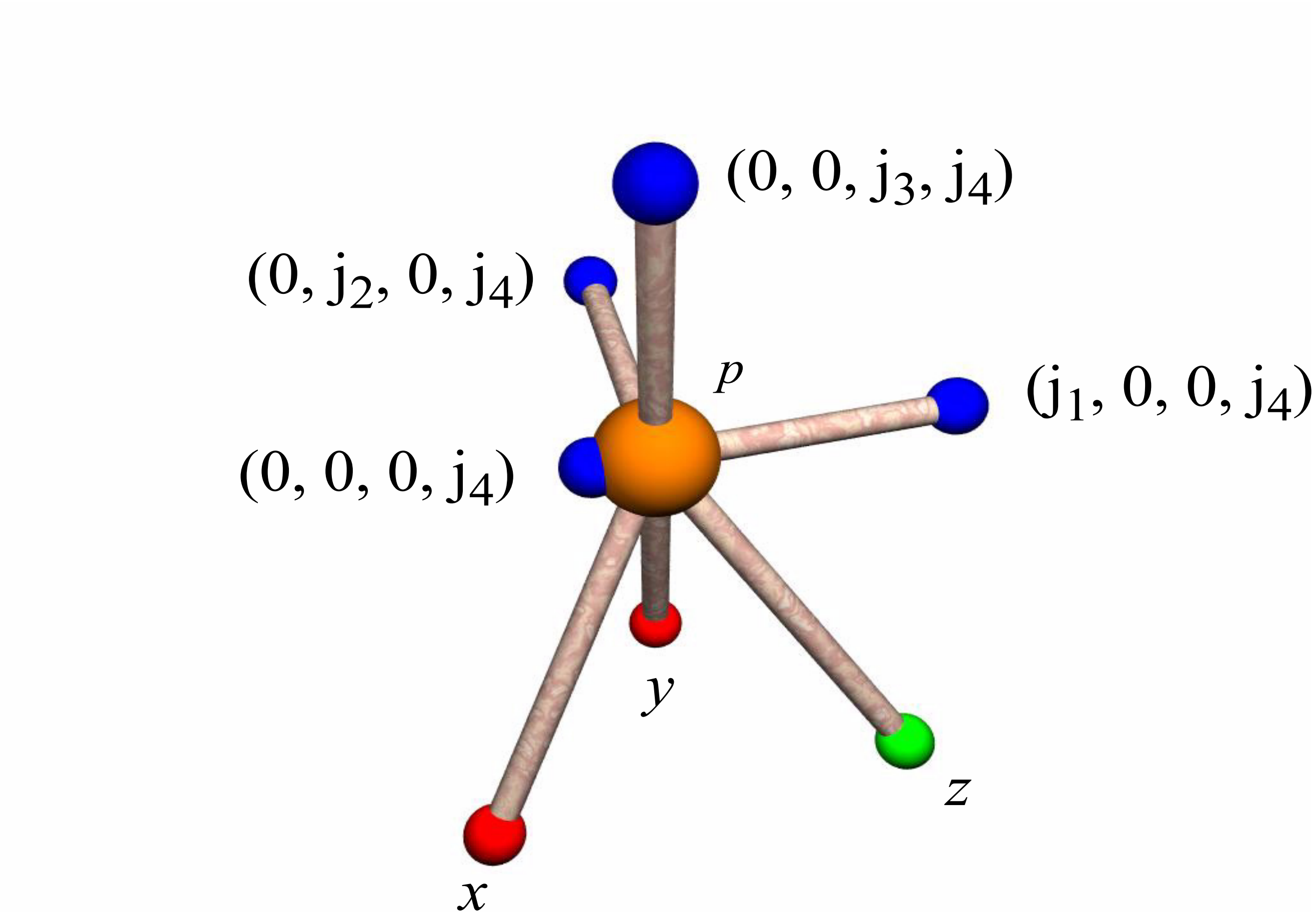
- Control jumps from p to $p + j$ with j computed as follows:
- If $\overrightarrow{z_{11} + p} < 0$ then $j_1 = t_1$, otherwise $j_1 = 0$;
- If $\overrightarrow{z_{11} + p} = 0$ then $j_2 = t_2$, otherwise $j_2 = 0$;
- If $\overrightarrow{z_{11} + p} > 0$ then $j_3 = t_3$, otherwise $j_3 = 0$;
- $j_4 = t_4$ unconditionally.

Totality

- The totality of transreal arithmetic allows the perspex machine to be universally deterministic, whereas the Turing machine can fail on non-deterministic programs.
- This makes the perspex machine super-Turing.
- The perspex machine can, nonetheless, give an exact emulation of the Turing machine by using deterministic perspex states to model non-deterministic states in a Turing machine.
- The perspex machine has no need of Turing's *oracle* or *choice machine,* because the perspex machine is total.

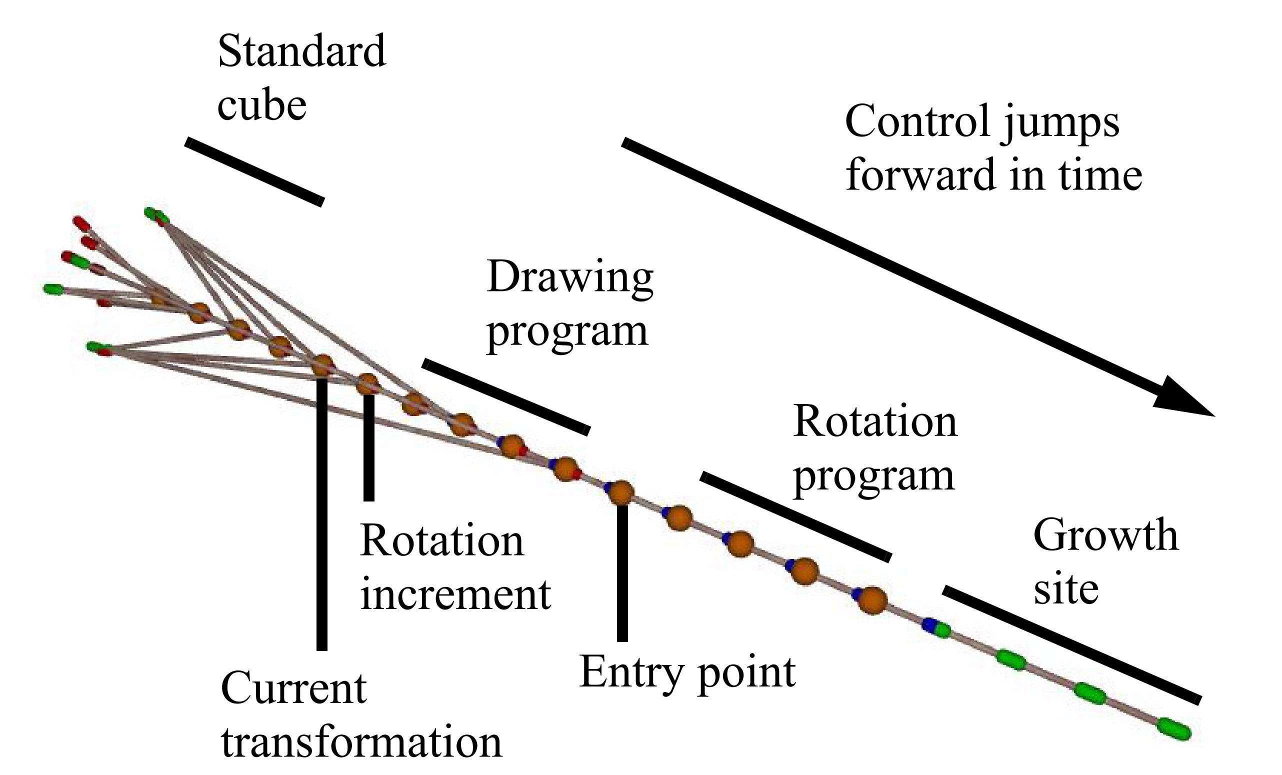
Perspex Neuron

The perspex instruction can be implemented as an artificial neuron stored at point *p* in space.



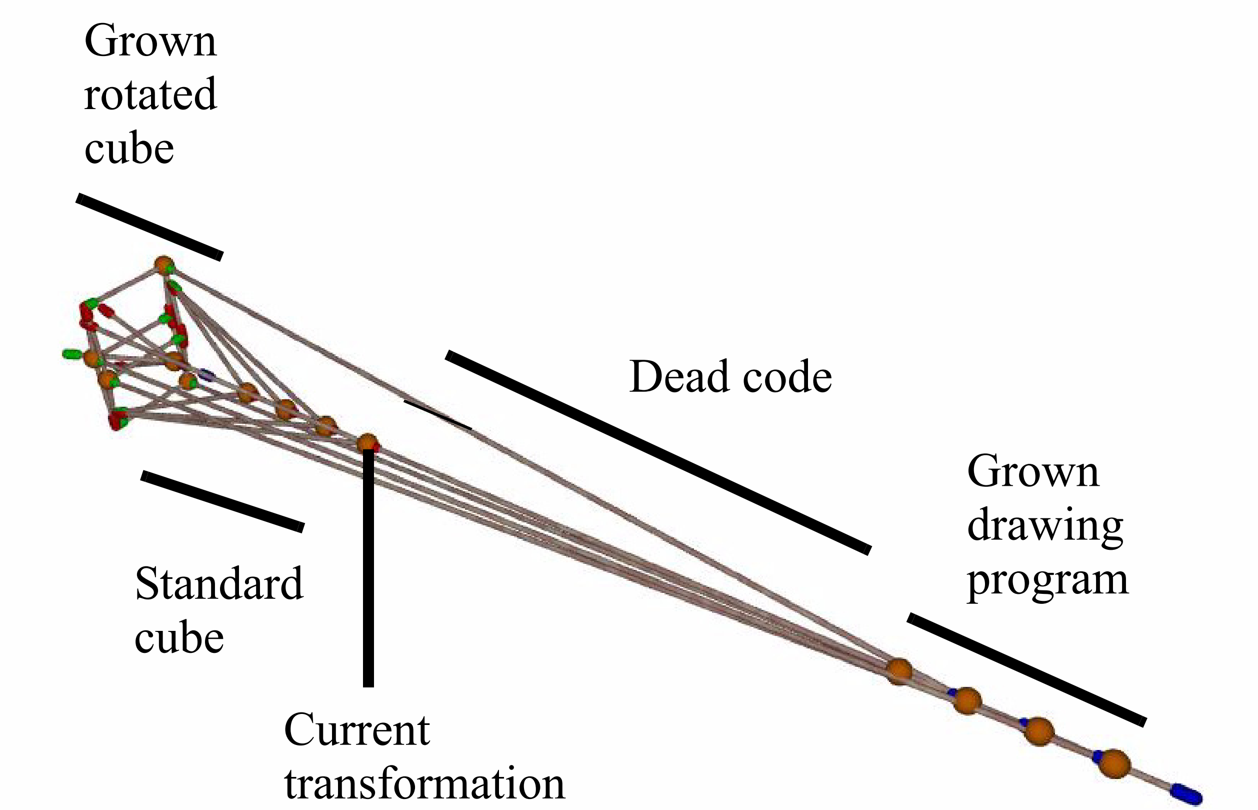
Perspex Programs

- Perspex programs look very much like networks of biological neurons.

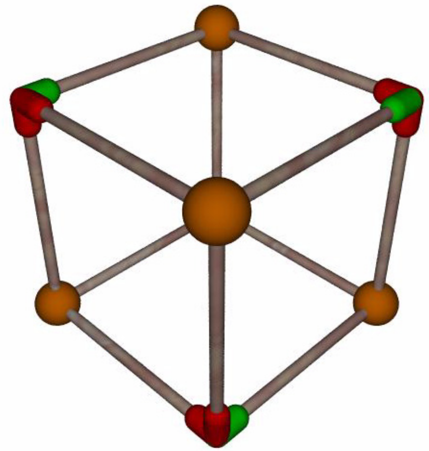


Perspex Programs

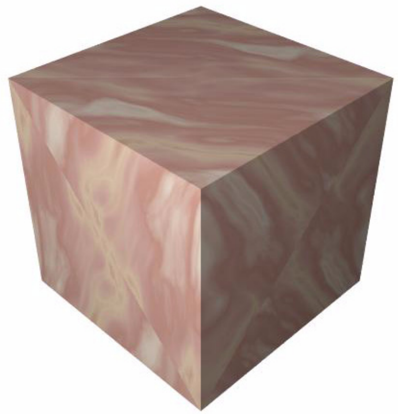
- Perspex programs grow by writing instructions and die by writing H.

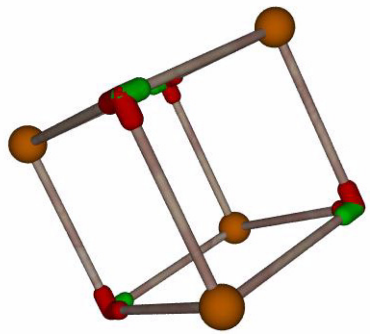


Perspex Rotation Program

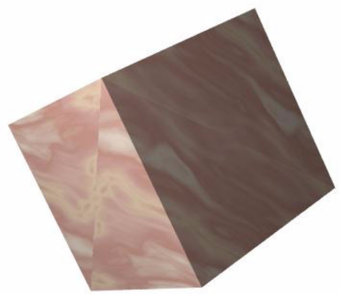


Standard cube





Rotated cube



Compilation

- The source code of any Turing program can be compiled automatically into a perspex neural network.
- Example C source code of Dijkstra's solution to the travelling salesman problem.
- Example perspex neural network compiled from the above source code.

Properties

- The perspex machine is continuous so any program can be interpolated. This:
- Supports a novel form of "genetic algorithm;"
- Allows programs to be filtered and re-constructed from their filter bands.
- In turn, this allows top-down, or "directed" genetic algorithms that search for a globally good solution in broad outline *before* finding the local optimum within that solution.
- In other words, evolution can appear as if it is directed.

Conclusion

- Infinity and nullity make arithmetic total.
- Transrational trigonometry is robust.
- The perspex machine is super-Turing.
- The perspex machine provides a direct description of: shapes, motions, computer programs, and neurons.
- Changing between these representations is trivial.
- The perspex machine allows global search and "directed" evolution.
- All programs can be compiled into perspex neural nets.

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