Transreal Basis for Paraconsistent Logic

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Agenda

- Advantages of transreal arithmetic as a consistent basis for paraconsistent logic
- Paraconsistent truth values
- Monotonicity
- Transreal space of all possible worlds
- Discriminant
- Value to science and society

Advantages

Paraconsistent Logic

- Allows reasoning over inconsistency
- Contains classical logic
- Potentially makes logical software more robust
- Potentially makes computer hardware robust to inconsistency!
- Potentially makes computer reasoning more like human reasoning

The Problem

Classical Inconsistency

ex contradictione quodlibet

1)	$p \& \neg p$	assumption
2)	p	from 1
3)	$\neg p$	from 1
4)	$p \vee q$	from 2
5)	q	from 3, 4
6)	$(p \& \neg p) \rightarrow q$	from 1,5

Explosiveness

- Classical logic explodes on a contradiction to encompass all syntactically possible conclusions - including inconsistent ones - in a chain reaction!
- Paraconsistent logic does not always explode on a contradiction; it generally allows only some conclusions to be drawn from a contradiction

Paraconsistent Logic

- Q: How is it possible to consistently axiomatise a paraconsistent logic that allows inconsistency?

Paraconsistent Logic

- Q: How is it possible to consistently axiomatise a paraconsistent logic that allows inconsistency?
- A: Transreal arithmetic is known to be consistent so translate paraconsistent logic into transreal arithmetic

Explosiveness

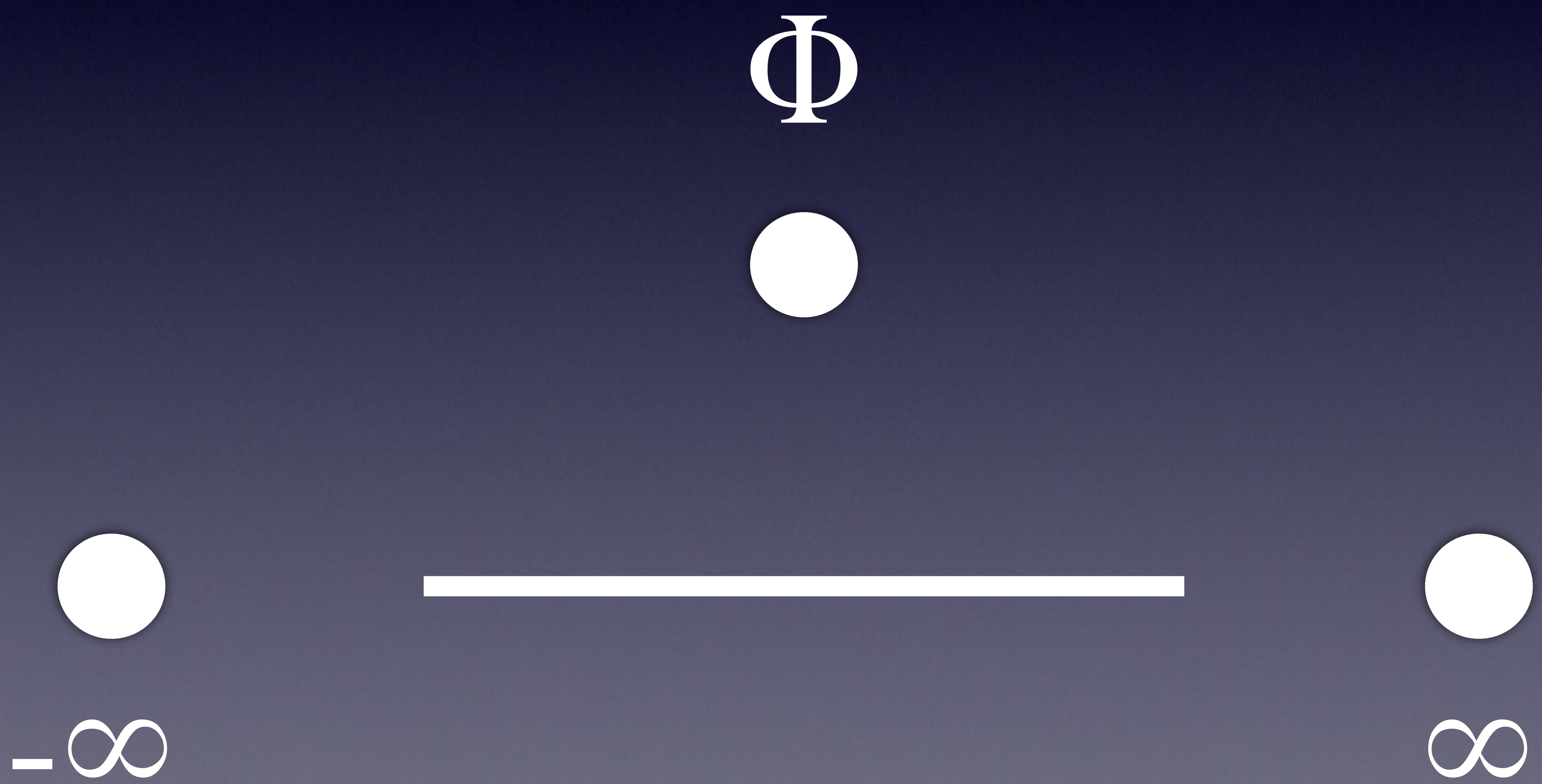
- Q: How is it possible to make a transreal logic generally non-explosive?

Explosiveness

- Q: How is it possible to make a transreal logic generally non-explosive?
- A: Use *min* and *max!*

The Solution

Transreal Number Line



Transreal Truth Values

- $-\infty$ models classical False
- $-\infty < v < 0$ models more False than True
- 0 models equally False and True
- $0 < v < \infty$ models more True than False
- ∞ models classical True
- Φ models Gap - no degree of False or True

Transreal Min

$$\min(a,b) = \begin{cases} a & : \quad a < b \\ a & : \quad a = b \\ a & : \quad b = \Phi \\ b & : \quad b < a \\ b & : \quad a = \Phi \end{cases}$$

Transreal Max

$$\max(a,b) = \begin{cases} a & : \quad a > b \\ a & : \quad a = b \\ a & : \quad b = \Phi \\ b & : \quad b > a \\ b & : \quad a = \Phi \end{cases}$$

Gap Values

- Logical statement: Frege's principle of compositionally - if any truth value in a possible world is Gap then all are Gap
- Transreal statement: Frege's principle of compositionally - if any truth value in a possible world is Φ then all are Φ

Transreal And, Or

$$a \,\&\, b = \begin{cases} \Phi & : \quad a = \Phi \text{ or } b = \Phi \\ \min(a,b) & : \quad \text{otherwise} \end{cases}$$

$$a \vee b = \begin{cases} \Phi & : \quad a = \Phi \text{ or } b = \Phi \\ \max(a,b) & : \quad \text{otherwise} \end{cases}$$

Transreal Negation

$$\neg a = -a$$

Transreal NAND

$$ab! = \neg(a \;\&\; b)$$

Transreal NAND

- Explosive for classical truth values $-\infty$, ∞
- Non-explosive for dialetheaic truth values, v, such that $-\infty < v < \infty$
- Non-explosive for Gap truth value, Φ
- Generalises all classical, truth functional logics to paraconsistent form
- Specifies a paraconsistent NAND gate

Logical Space

- Propositions are trans-Cartesian axes
- A possible world is a point in space
- The space of all possible worlds is the whole of trans-Cartesian space

Transformations

- Transformations operate identically on transreal logic and trans-Cartesian space

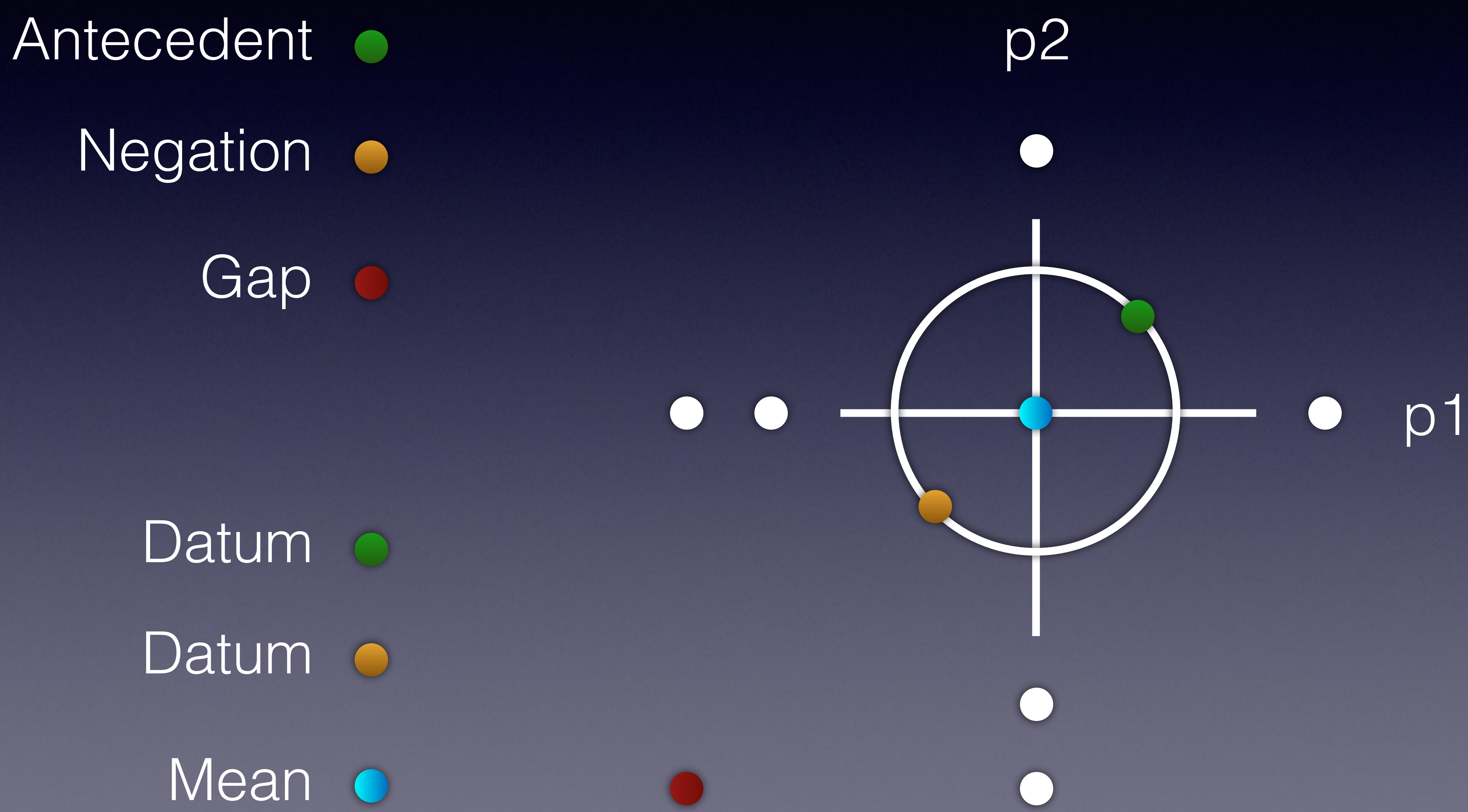
Monotonicity

- Logical statement of monotonicity:
  a conclusion departs no further from equally False and True than the most extreme of its antecedents

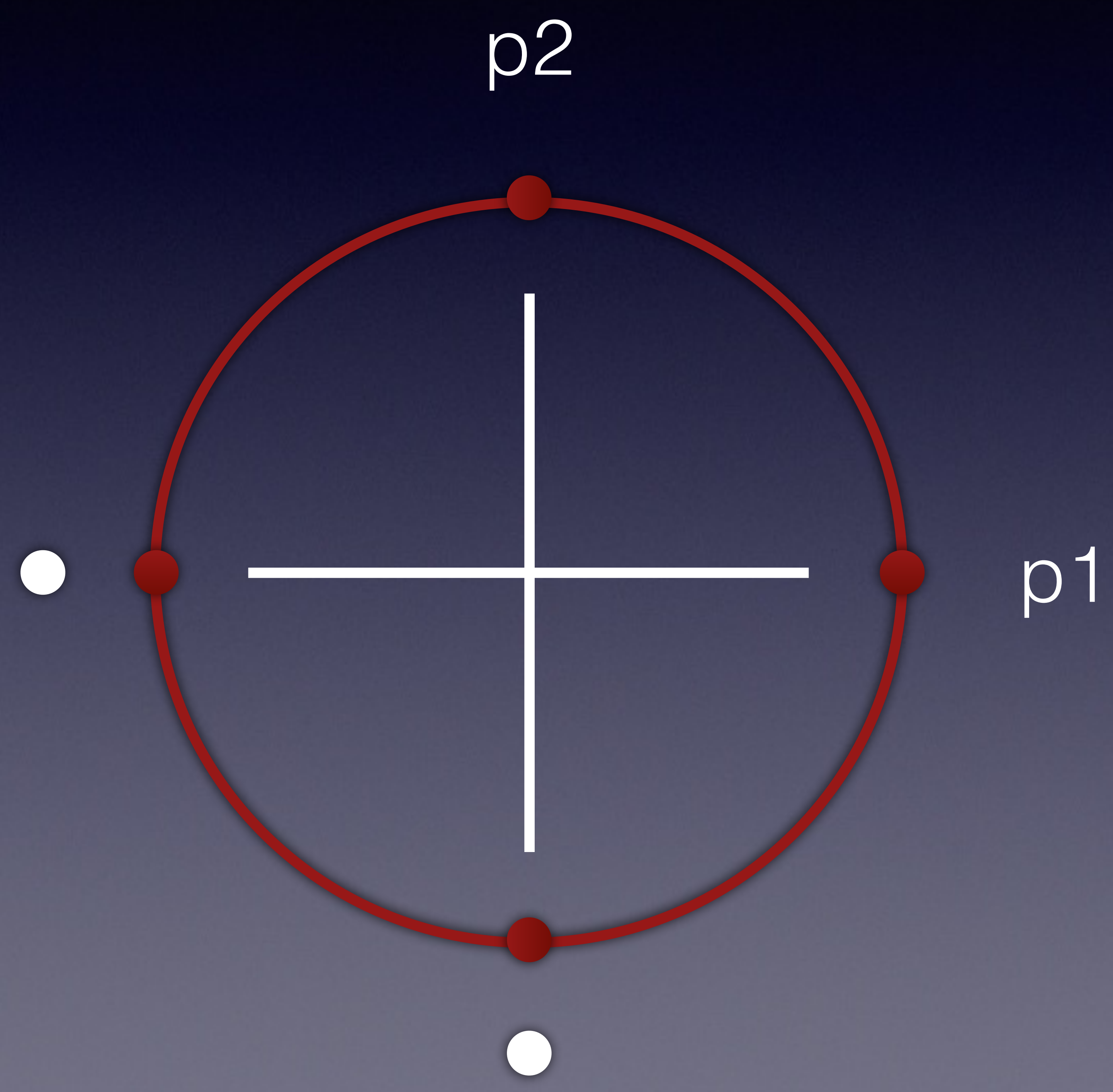
- Spatial statement of monotonicity:
  a conclusion does not lie outside a sphere, centred on the origin, that most tightly bounds its antecedents

Trans-Cartesian Axis

Logical Space



Explosions Only at Infinity



Transmetric

- $|a, b| = 0$ when $a = b$
- $|a, b|$ is the ordinary metric, calculated with transreal arithmetic, when $a \neq b$
- Here we use Euclidean distance as the ordinary metric
- $|a| = |a, 0|$

Determinant

- The determinant, d, in the range $0 \leq d \leq \infty$ or $d = \Phi$, classifies each predicate, p, in paraconsistent, transreal logic as classical, dialetheaic, or gappy and it measures the degree to which it is dialetheaic

$$d = |p, \neg p|$$

Value

Science and Society

- Transreal arithmetic is a consistent basis for paraconsistent logic
- Nullity models Gap values in modern logics
- The determinant classifies paraconsistent propositions - for the first time!
- Trans-Cartesian space instantiates Wittgenstein’s logical space - for the first time!

Science and Society

- Potentially makes logical software more robust
- Potentially makes computer hardware robust to inconsistency!
- Potentially makes computer reasoning more human

This is the first transreal logic