Construction of the Transcomplex Numbers

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Agenda

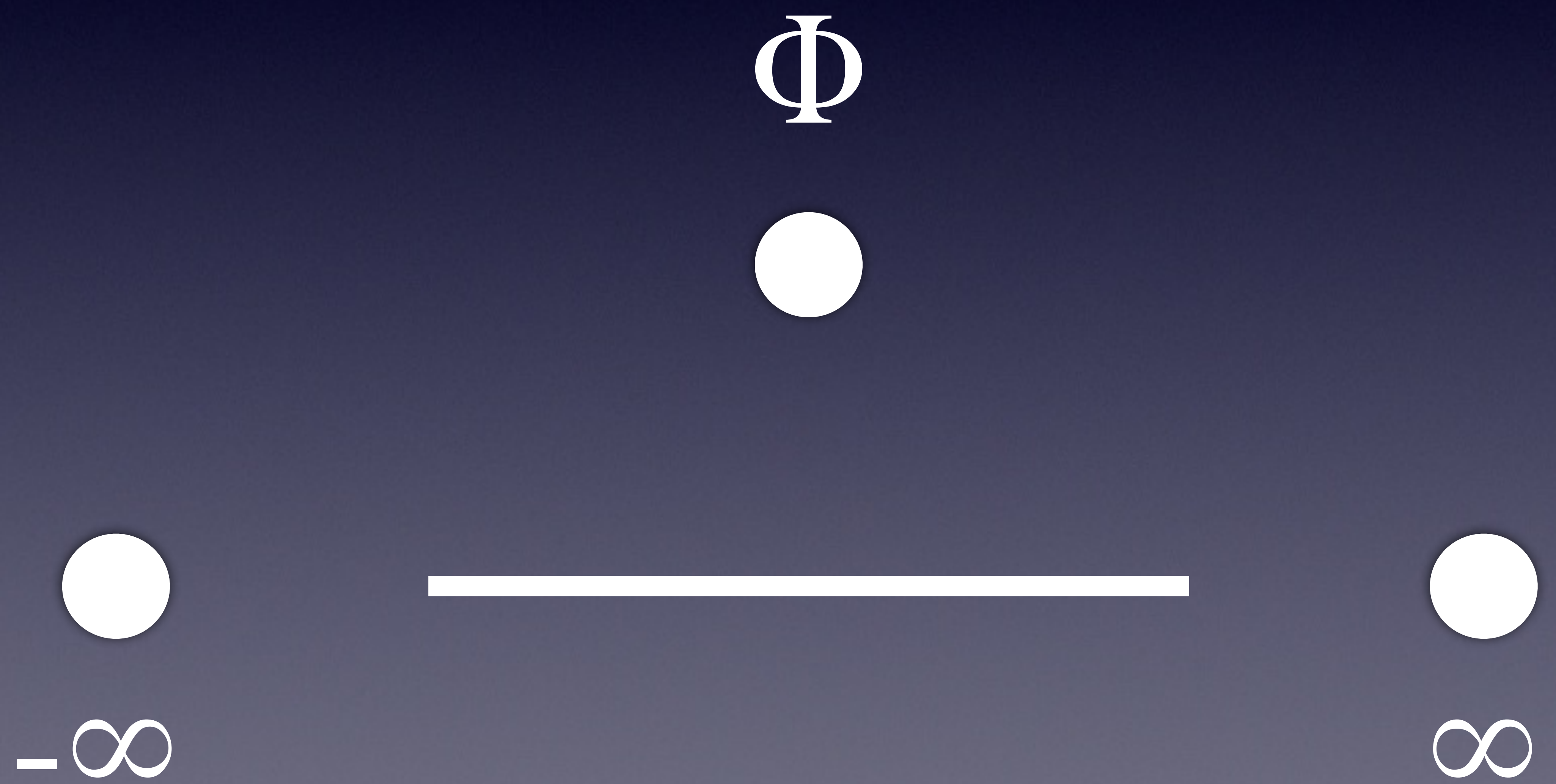
- Advantages of transcomplex numbers
- Containment relationships
- Construction of the transcomplex numbers
- Value to science and society

Advantages

- Transcomplex arithmetic builds on the foundation of transreal arithmetic
- Consistency of transcomplex and transreal numbers proved by construction from the complex numbers
- Transcomplexes allow the solution of mathematical and physical problems at singularities
- Transcomplexes make mathematical software more reliable

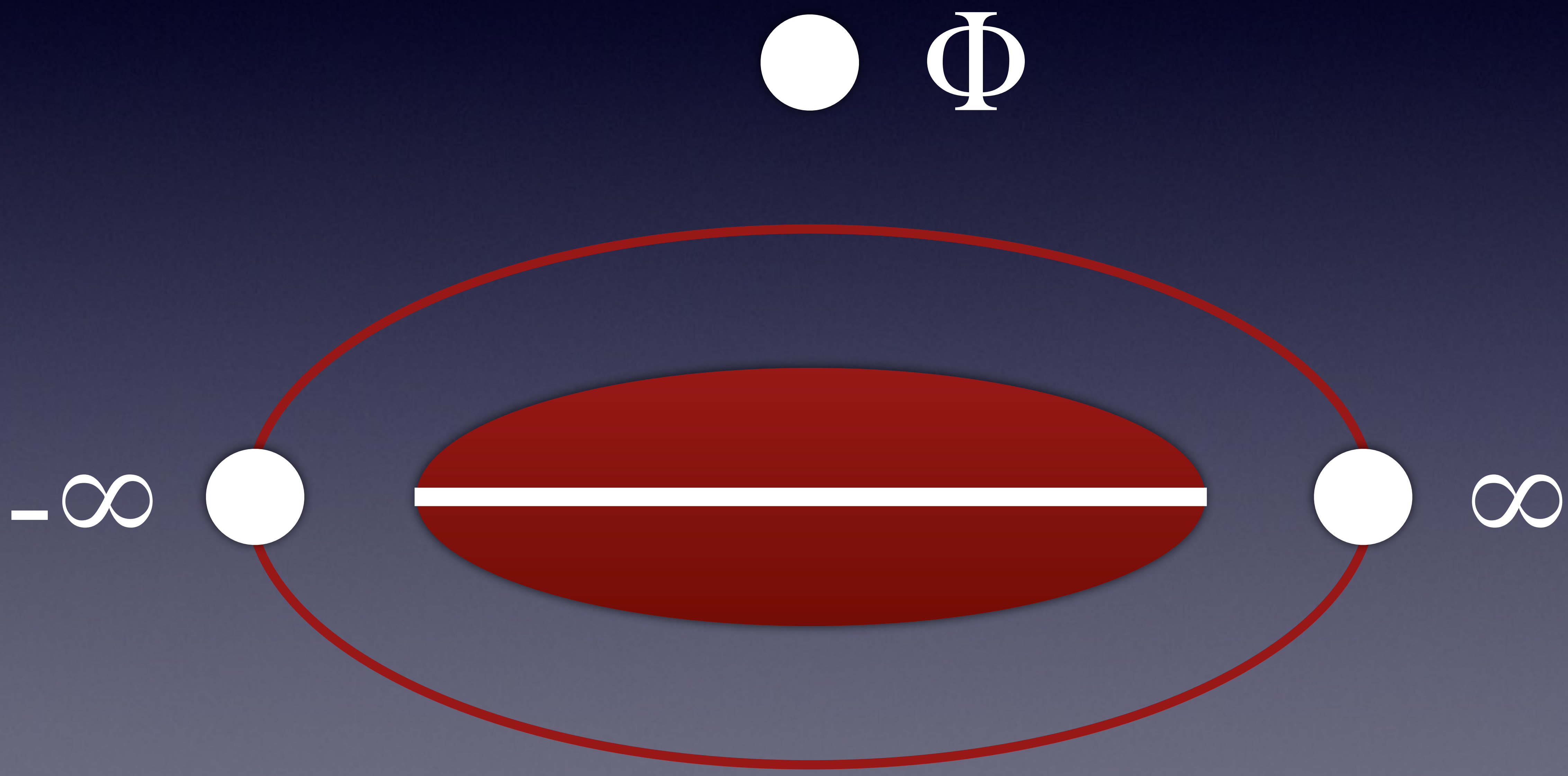
Overview

Transreal Number Line

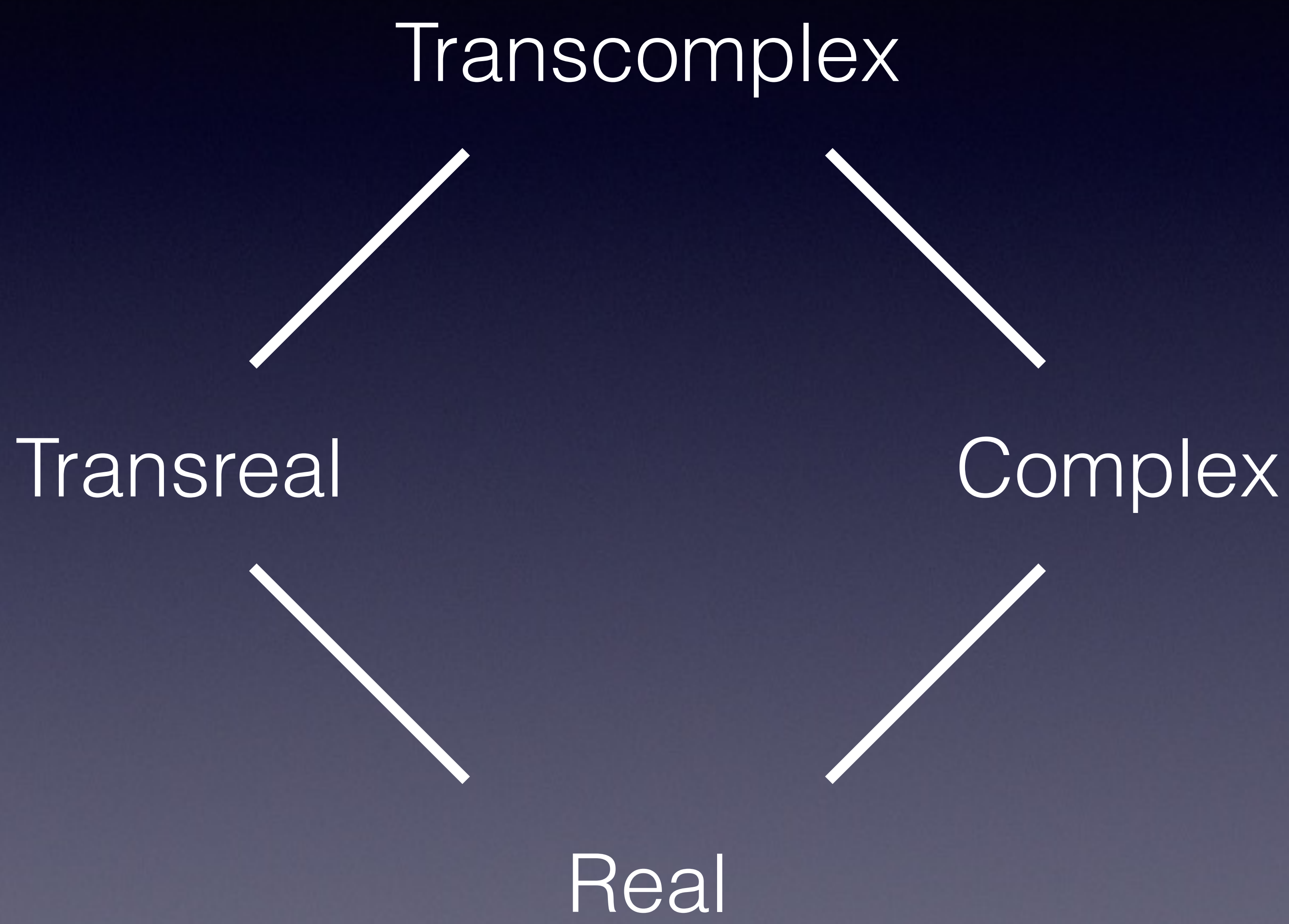


Transcomplex Plane

Revolution of the transreal number line



Containment



Construction

- Aims to give sense to dividing complex numbers by zero

Construction

$$T \coloneqq \left\{ (x, y);\ x \in \mathbb{C},\ y \in \{0,1\} \right\}$$

Construction

$$T := \left\{(x,y);\ x \in \mathbb{C},\ y \in \{0,1\}\right\}$$

$$(x,y) \sim (w,z)$$

$$\Updownarrow$$

$$\exists \alpha \in \mathbb{R}^+;\ x = \alpha w \quad \text{and} \quad y = \alpha z$$

Construction

Reflexive

$$(x, y) \sim (x, y)$$

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Symmetric

$$(x,y) \sim (w,z) \Rightarrow (w,y) \sim (x,y)$$

Construction

Reflexive

$$(x,y) \sim (x,y)$$

Symmetric

$$(x,y) \sim (w,z) \Rightarrow (w,y) \sim (x,y)$$

Transitive

$$(x,y) \sim (w,z) \text{ and } (w,z) \sim (u,v) \Rightarrow (x,y) \sim (u,v)$$

Construction

$$\mathbb{C}^T \coloneqq T/{\sim}$$

Construction

Addition

If $[x,y],[w,z] \in \left\{[u,0]; u \in \mathbb{C} \setminus \{0\}\right\}$:

$$[x,y]+[w,z] \coloneqq \left[\frac{x}{|x|}+\frac{w}{|w|},0\right]$$

otherwise,

$$[x,y]+[w,z] \coloneqq [xz+wy,yz]$$

Construction

Multiplication

$$[x, y] \times [w, z] := [xw, yz]$$

Construction

Multiplication

$$[x,y]\times[w,z] \coloneqq [xw,yz]$$

Opposite

$$-[x,y] \coloneqq [-x,y]$$

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Reciprocal

$$[x,y]^{-1} \coloneqq \begin{cases} \left[\frac{y}{x},1\right], & x \neq 0 \\ [y,x], & x = 0 \end{cases}$$

Construction

Subtraction

$$[x,y]-[w,z]\coloneqq[x,y]+\left(-[w,z]\right)$$

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Subtraction

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Division

$$[x,y]\div[w,z]\coloneqq[x,y]\times[w,z]^{-1}$$

Construction

$$\mathbb{C}^T = \left\{[x,1]; x \in \mathbb{C}\right\} \cup \left\{[w,0]; w \in \mathbb{C}, |w| = 1\right\} \cup \left\{[0,0]\right\}$$

Construction

$\{[x,1]; x \in \mathbb{C}\}$ is a copy of $\mathbb{C}$

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$$x = [x,1]$$

$$[x,y] = \frac{x}{y}$$

Construction

(r,θ)

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$$(r,\theta) = \begin{cases} [x,1] \\ \\ [u,0] \\ \\ [0,0] \end{cases}$$

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Construction

$$\mathbb{C}^T = \mathbb{C} \bigcup \left\{(\infty,\theta);\, \theta \in (-\pi,\pi]\right\} \bigcup \{\Phi\}$$

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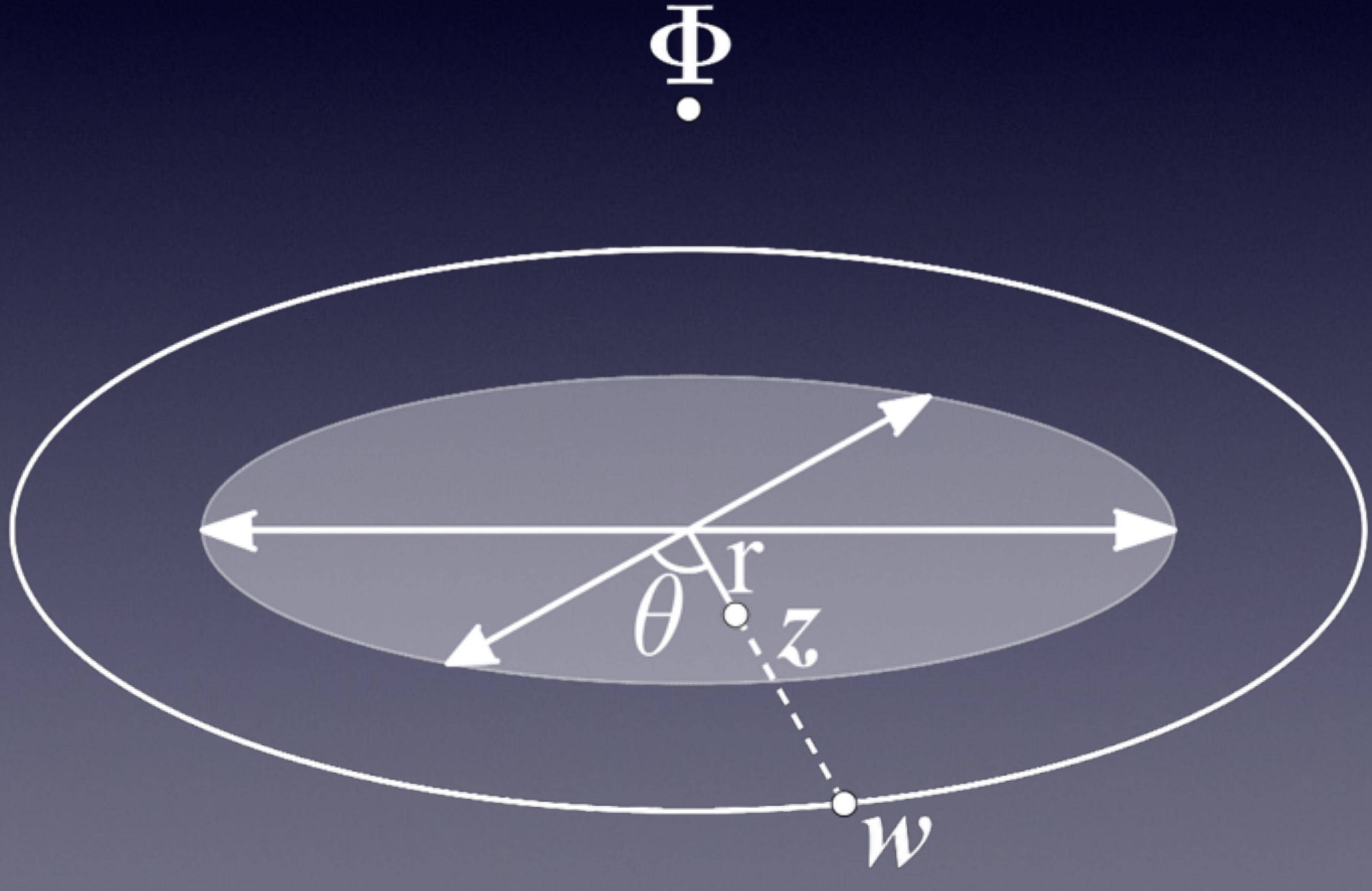

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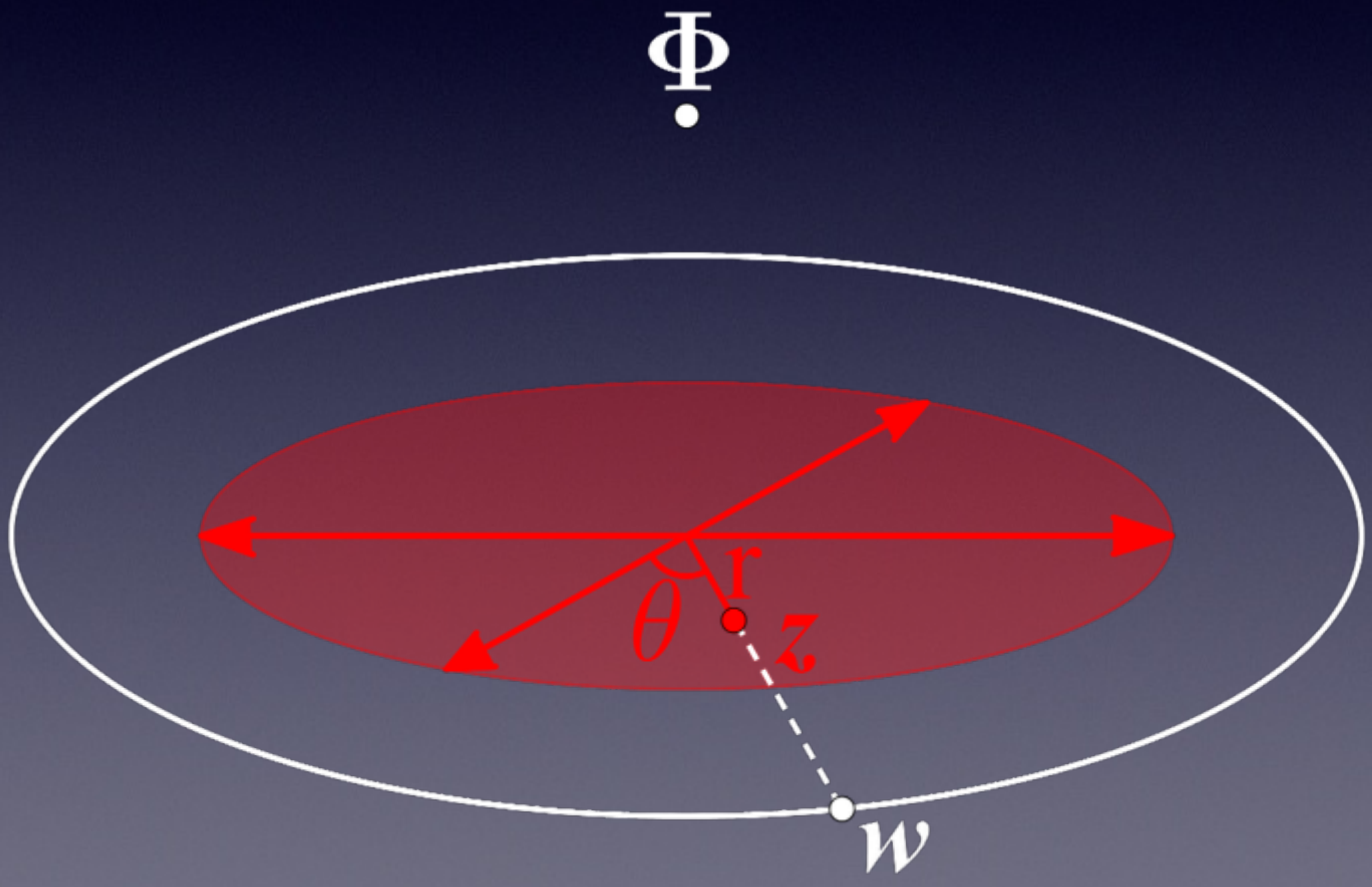

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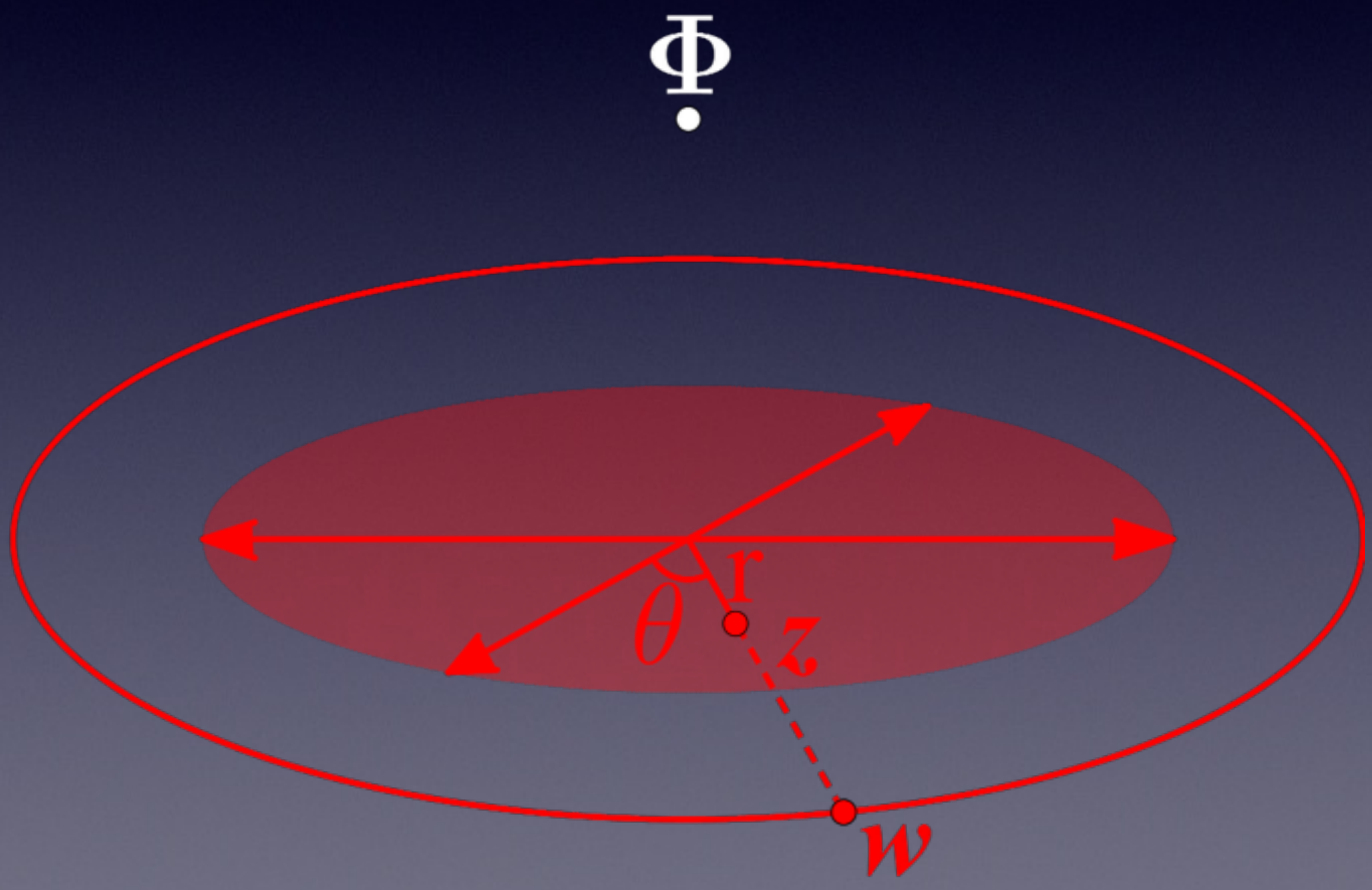

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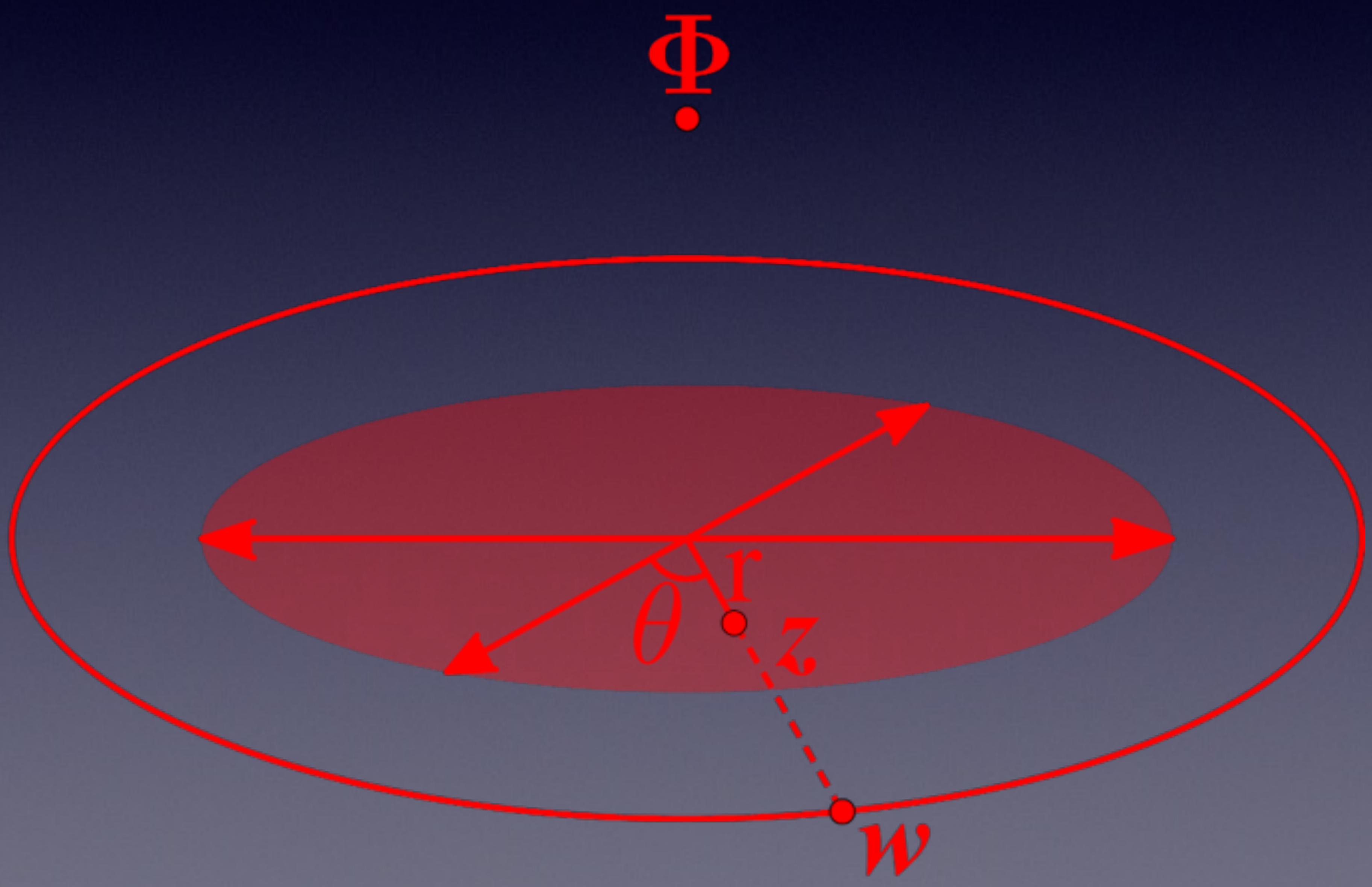

Examples

$$
\begin{aligned}
1 \div 0 &= [1,1] \div [0,1] = [1,1] \times [0,1]^{-1} \\
&= [1,1] \times [1,0] = [1 \times 1, 1 \times 0] \\
&= [1,0]
\end{aligned}
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polar

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$$= [0,0] = (\Phi, 0)$$

polar

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&= [i,1] \times [1,0] &&= [i \times 1, 1 \times 0] \\
&= [i,0] &&= \left(\infty, \frac{\pi}{2}\right)
\end{aligned}
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polar

Value

Reach and Reliability

- Construction proves the consistency of the transcomplex and transreal numbers from the complex numbers
- Transcomplex numbers allow the solution of mathematical and physical problems at singularities
- Transcomplex numbers make mathematical software more reliable

Transcomplexes
are at the beginning
of their development