A Totally New Supercomputer

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Disruption

Disruptive technologies allow new entrants to challenge the market and earn a very great deal of money:

- The market can only go to the new entrant for the new technology
- Competitors are blind-sided and don't know how to respond to the new entrant
- Market turmoil generates publicity for the new entrant
- With luck and investment the new entrant becomes established as the market leader

But, first, you need a disruptive technology …

Disruption

The disruptive technology should sound palatable:

> *Total supercomputers exploit a new kind of arithmetic to achieve very high reliability and performance. Ordinary arithmetic is partial, it fails when some operations are carried out on certain numbers, but a new class of total arithmetics means that arithmetical operations always succeed. This makes total computers highly reliable. And reliable computers can be pushed harder so that they deliver blistering performance*

But it should pack a punch!

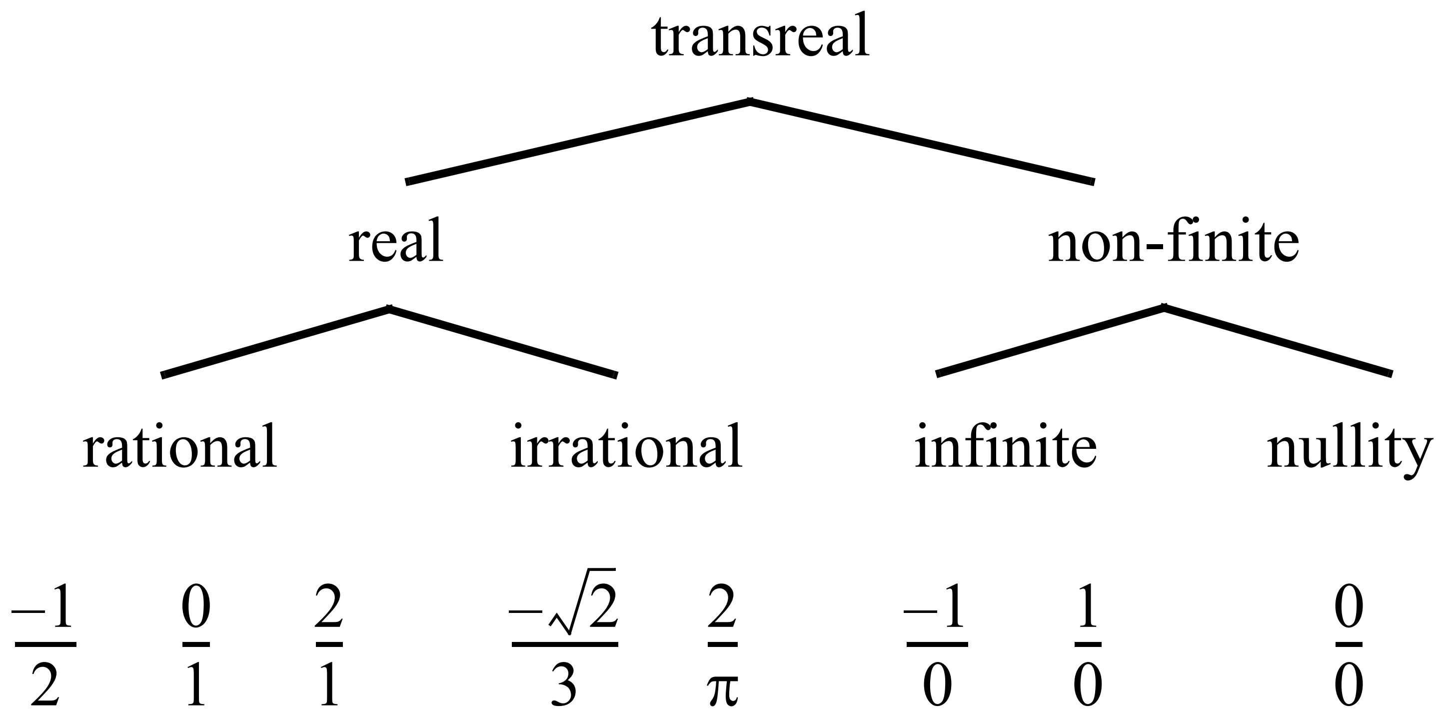
Disruption

The operations of a total arithmetic work on any numbers. In particular, total arithmetics allow division by zero!

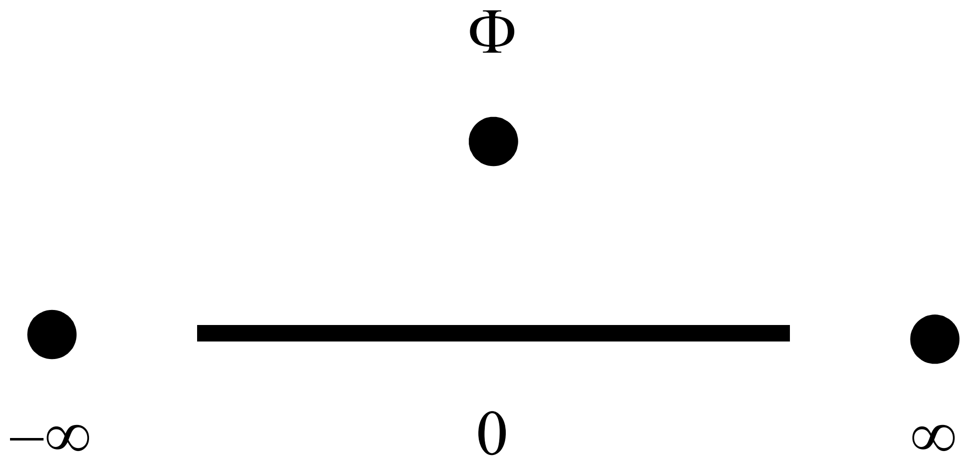
- How to Divide Arithmetically by Zero
- Mathematical Consequences for Physics
- Mathematical Consequences for Computing
- Physical Consequences for Computing
- Consequences for Society
- Consequences for Business

Transreal Numbers

The transreal numbers are all of the real numbers, as used in everyday life, together with three non-finite numbers: $-\infty = \frac{-1}{0}$, $\Phi = \frac{0}{0}$, $\infty = \frac{1}{0}$



Transreal Numbers



- Positive infinity, ∞, is the biggest transreal number
- Negative infinity, –∞, is the smallest transreal number
- Nullity, Φ, is the only transreal number that is not negative, not zero, and not positive

Transreal Numbers

- Positive Infinity, ∞, is any positive number divided by zero

  Its standard form is $\infty = \dfrac{1}{0}$

- Negative infinity, $-\infty$, is any negative number divided by zero

  Its standard form is $-\infty = \dfrac{-1}{0}$

Transreal Numbers

- Nullity, Φ, is zero divided by zero

  Its standard form is $\Phi = \frac{0}{0}$

- The fraction zero, 0, is the integer zero, 0, divided by any positive or negative number

  Its standard form is $0 = \frac{0}{1}$

Transreal Fractions

A *transreal number* is a *transreal fraction* of the form $\frac{n}{d}$, where:

- n is the *numerator* of the fraction
- d is the *denominator* of the fraction
- n, d are *real numbers (or transreal numbers)*
- $d \geq 0$
- Examples: $\frac{-1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{-\pi}{2}, \frac{-1}{2\pi}, \frac{1}{2\pi}, \frac{-1}{0}, \frac{1}{0}, \frac{0}{0}$

Transreal Fractions

- An *improper transreal fraction*, $\frac{n}{-d}$, may have a negative denominator, $-d < 0$
- An improper transreal fraction is converted to a *proper transreal fraction* by negating both the numerator and the denominator
- Example: $\frac{2}{-3} = \frac{-2}{-(-3)} = \frac{-2}{3}$
- Example: $\frac{0}{-1} = \frac{-0}{-(-1)} = \frac{0}{1}$

Transreal Multiplication

Two *proper transreal fractions* are multiplied like this:

- $\dfrac{a}{b} \times \dfrac{c}{d} = \dfrac{a \times c}{b \times d}$
- Example: $3 \times \infty = \dfrac{3}{1} \times \dfrac{1}{0} = \dfrac{3 \times 1}{1 \times 0} = \dfrac{3}{0} = \infty$
- Example: $0 \times \infty = \dfrac{0}{1} \times \dfrac{1}{0} = \dfrac{0 \times 1}{1 \times 0} = \dfrac{0}{0} = \Phi$
- Example: $-3 \times \infty = \dfrac{-3}{1} \times \dfrac{1}{0} = \dfrac{-3 \times 1}{1 \times 0} = \dfrac{-3}{0} = -\infty$

Transreal Division

Two *proper transreal fractions* are divided like this:

- $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$
- Example: $\infty \div 3 = \frac{1}{0} \div \frac{3}{1} = \frac{1}{0} \times \frac{1}{3} = \frac{1 \times 1}{0 \times 3} = \frac{1}{0} = \infty$
- Example:

$$\infty \div (-3) = \frac{1}{0} \div \frac{-3}{1} = \frac{1}{0} \times \frac{1}{-3} = \frac{1}{0} \times \frac{-1}{-(-3)}$$

$$= \frac{1}{0} \times \frac{-1}{3} = \frac{1 \times (-1)}{0 \times 3} = \frac{-1}{0} = -\infty$$

Transreal Addition

Two *proper transreal fractions* are added like this:

- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, except that:
- $(\pm\infty) + (\pm\infty) = \frac{\pm 1}{0} + \frac{\pm 1}{0} = \frac{(\pm 1) + (\pm 1)}{0}$

Transreal Addition

- $(\pm\infty) + (\pm\infty) = \frac{\pm 1}{0} + \frac{\pm 1}{0} = \frac{(\pm 1) + (\pm 1)}{0}$

Examples:

- $\infty + \infty = \frac{1}{0} + \frac{1}{0} = \frac{1 + 1}{0} = \frac{2}{0} = \infty$
- $(-\infty) + (-\infty) = \frac{-1}{0} + \frac{-1}{0} = \frac{(-1) + (-1)}{0} = \frac{-2}{0} = -\infty$
- $\infty + (-\infty) = \frac{1}{0} + \frac{-1}{0} = \frac{1 + (-1)}{0} = \frac{0}{0} = \Phi$

Transreal Addition

- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

Examples:

- $\frac{2}{3} + \infty = \frac{2}{3} + \frac{1}{0} = \frac{2 \times 0 + 3 \times 1}{3 \times 0} = \frac{3}{0} = \infty$
- $\frac{2}{3} + \Phi = \frac{2}{3} + \frac{0}{0} = \frac{2 \times 0 + 3 \times 0}{3 \times 0} = \frac{0}{0} = \Phi$
- $\frac{2}{3} + \frac{4}{5} = \frac{2 \times 5 + 3 \times 4}{3 \times 5} = \frac{22}{15}$

Transreal Subtraction

Two *proper transreal fractions* are subtracted like this:

- $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$

Examples:

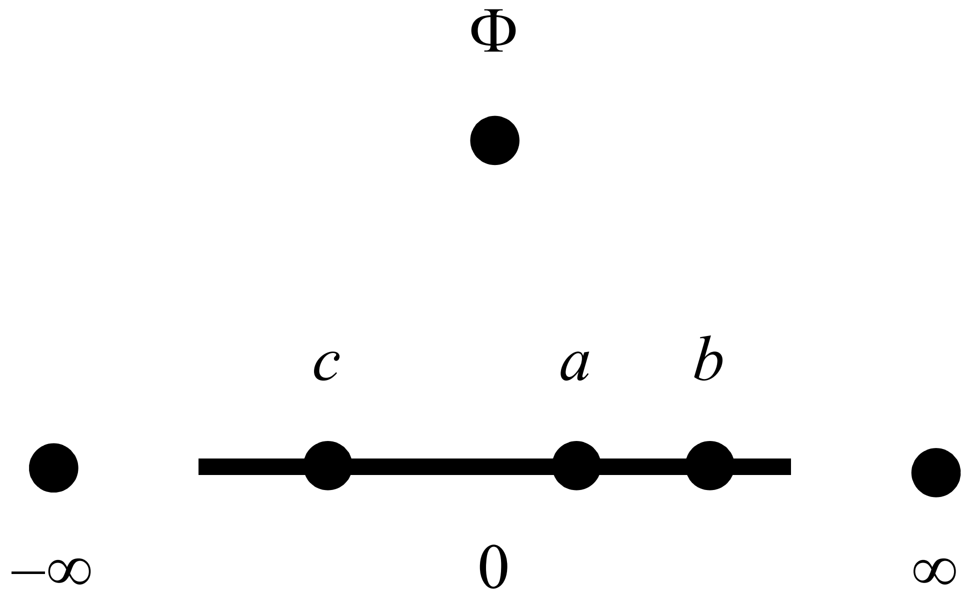
- $\infty - \infty = \frac{1}{0} - \frac{1}{0} = \frac{1}{0} + \frac{-1}{0} = \frac{1 + (-1)}{0} = \frac{1 - 1}{0} = \frac{0}{0} = \Phi$
- $\frac{1}{2} - \frac{3}{5} = \frac{1}{2} + \frac{-3}{5} = \frac{(1 \times 5) + (2 \times (-3))}{2 \times 5} = \frac{5 + (-6)}{10}$
  $= \frac{-1}{10}$

Transreal Arithmetic

- Transreal arithmetic is consistent
- Transreal arithmetic contains real arithmetic
- Transreal arithmetic extends to transcomplex arithmetic
- Transreal arithmetic extends to transquaternion arithmetic
- Transreal arithmetic extends to transoctonion arithmetic
- Transreal arithmetic extends many other arithmetics

Physics - Linking Hypothesis

First, we need to link mathematics to physics



Physics - Linking Hypothesis

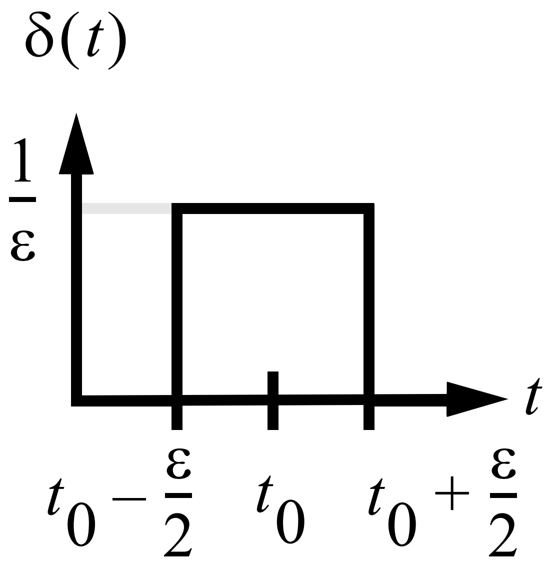
- Any real-numbered, physical quantity, a, can move continuously to a physical quantity, b, that lies between a and ∞
- Similarly, any real-numbered, physical quantity, a, can move continuously to a physical quantity, c, that lies between a and $-\infty$
- In particular, a can feel infinite positive and negative forces that cause it to move classically
- I hypothesise that quantum physics works similarly

Physics - Linking Hypothesis

- There is no quantity between Φ and any real-numbered or transreal-numbered quantity, a, so there is no continuous (classical) motion between a and Φ
- In particular, a cannot feel a nullity force that causes it to move classically
- I hypothesise that quantum physics works similarly

Physics - Example

- The Dirac Delta is used to transfer an action asymptotically quickly, but not instantaneously



- Feynman wanted to exempt a moving electron from acting on itself via the electric field, but couldn't

Physics - Example

- If we assume that the interaction of the electric field with a distant electron is asymptotically fast, because the field perturbation has to cross the electron, then the Dirac Delta transfers the action between the electric field and the distant electron in the usual way

- If we assume that the interaction of the electric field with an electron itself is instantaneous, because the field perturbation does not have to cross the electron, then the Dirac Delta collapses to a Box function with an area of nullity so, by our linking hypothesis, the electron does not feel a force from the electric field, nor does the electric field feel a force from the electron

Physics - Example

- When epsilon is exactly zero, $\varepsilon = 0$, we have width exactly zero, $w = \varepsilon = 0$, and height exactly infinity, $h = 1/\varepsilon = 1/0 = \infty$. Whence the area, a, is exactly nullity, $a = w \times h = 0 \times \infty = (0/1) \times (1/0) = (0 \times 1)/(1 \times 0) = 0/0 = \Phi = 0/0 = \varepsilon/\varepsilon \neq 1$
- This gives Feynman the behaviour he wanted, but it does it by assuming that the universe operates according to transreal arithmetic not real arithmetic

Physics - Metric Properties

Metric spaces are generalised to transmetric spaces by replacing *greater-than-or-equals* with *not-less-than* in the axioms of these topological spaces

Metric (m)	Transmetric (t)
$m(a,b) = m(b,a)$	$t(a,b) = t(b,a)$
$m(a,b) \geq 0$	$t(a,b) \nless 0$
$m(a,b) = 0 \Leftrightarrow a = b$	$t(a,b) = 0 \Leftrightarrow a = b$
$m(a,b) + m(b,c) \geq m(a,c)$	$t(a,b) + t(b,c) \nless t(a,c)$

Physics - Metric Properties

Transmetric spaces:

- Generalise all topologies relevant to physics
- Generalise calculus
- Generalise trigonometry
- Generalise geometry
- They work by using a generalised arithmetic

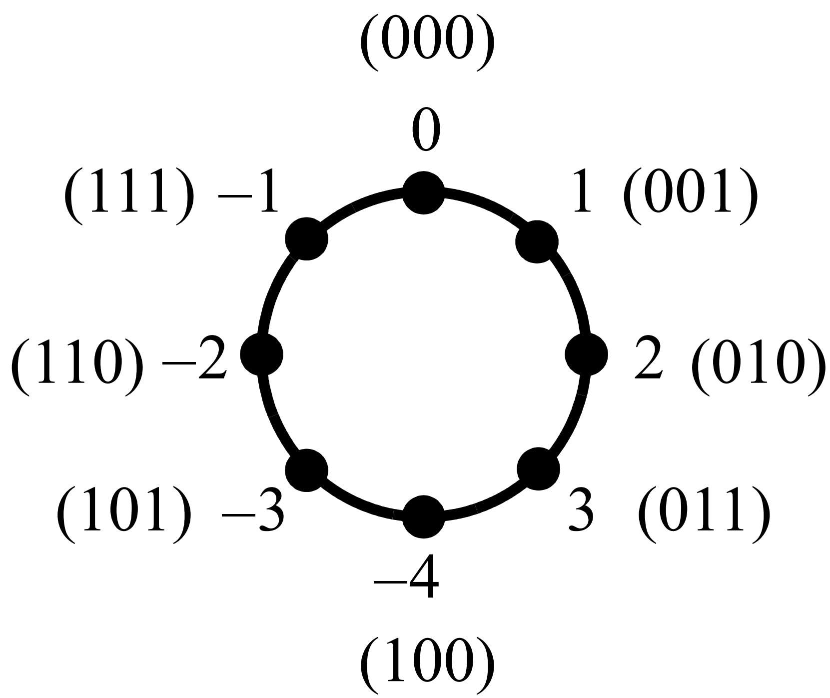
In other words, the mathematical tools used in physics can be generalised so that they deal correctly with division by zero

Physics

- If the universe operates according to ordinary arithmetic then there must be a physical censor that outlaws division by zero and all of its infinitely many mathematical consequences
- If the universe operates according to transarithmetic then it can divide by zero
- As transarithmetic supports an infinitely simpler explanation of the universe than ordinary arithmetic, Occam’s razor says that we should abandon ordinary arithmetic as a model of physics and use transarithmetic instead

Computer Arithmetic

Here is a 3-bit, two's complement encoding of the integers. The codes are shown in parentheses, e.g. (000)



Computer Arithmetic

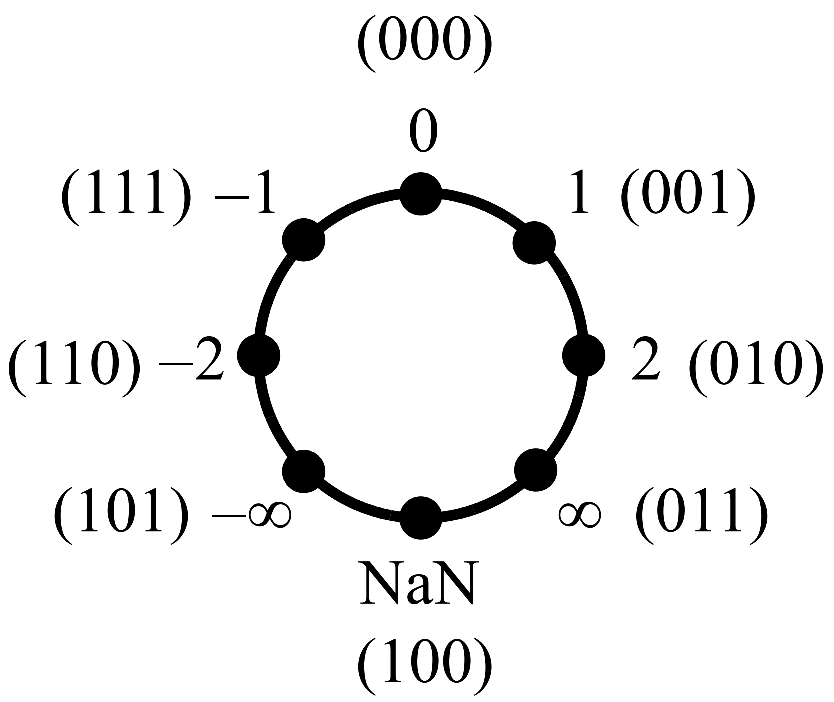
The most negative number, n, is called the *weird* number because it has the property that $-n = n < 0$. In particular, $n^2 < 0$. This destroys the usual meaning of direction and distance

- Most computers use two's complement arithmetic with the weird number

- Some computer language standards make this bug mandatory – every version of the language must misbehave in the same way

Computer Arithmetic

Some computers saturate arithmetic at positive infinity and negative infinity, and have a single object which is *N*ot *a N*umber - NaN



Computer Arithmetic

- There is no international standard for integer NaN
- If a computer supports integer NaN it may well behave differently on a different computer
- There is an international standard for floating-point NaN
- 80-bit floating-point arithmetic has eighteen quintillion objects which are NaN
- Integer NaN and floating-point NaN behave differently

Computer Arithmetic

- Floating point NaN has the property NaN ≠ NaN
- NaN ≠ NaN destroys the usual meaning of direction and distance and identity
- NaN cannot be used as a value in physics

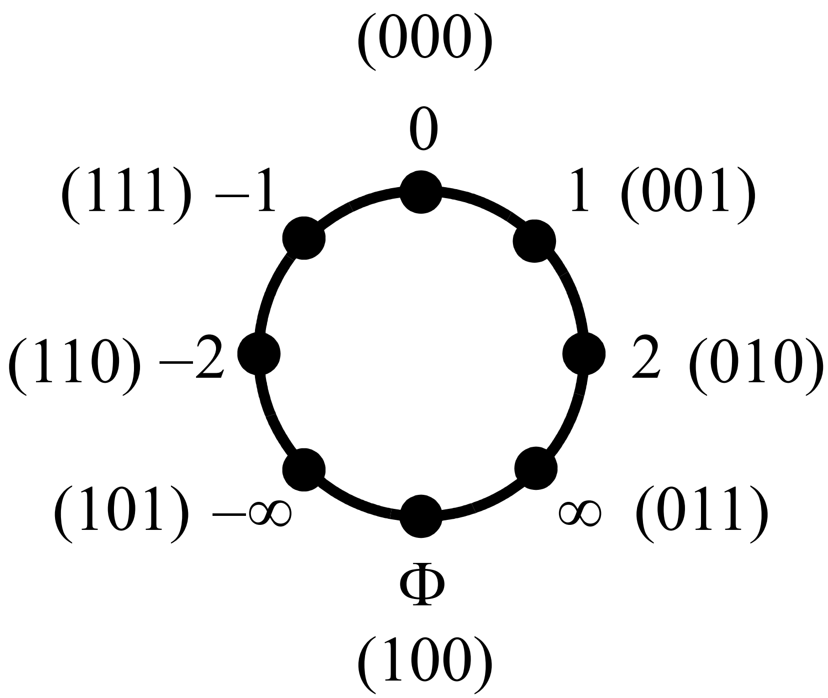
Computer Arithmetic

The international standard for floating-point arithmetic mandates that some syntactically correct sentences are semantic errors

- "$a > b$" is syntactically (grammatically) correct
- But "$a > b$" generates an exception (error) if any of a, b is NaN
- The standard mandates that all computer languages contain these errors, but chip designers, language designers, and end-user programmers can choose how to handle the errors

Computer Arithmetic

Here is the two's complement encoding of transintegers



Computer Arithmetic

- Transinteger $-\infty$, ∞, Φ behave in exactly the same way as floating-point $-\infty$, ∞, Φ
- Φ **is not** a weird number because it **does not** have the property that $-n = n < 0$ **nor** the property $n^2 < 0$
- Transreal $-\infty$, ∞, Φ preserve the usual meaning of direction and distance (with the addition of a distance Φ that is consistent with all of the other distances) and preserve identity
- Transreal $-\infty$, ∞, Φ can be used as values in physics

Computer Arithmetic

- Every syntactically correct sentence of transreal arithmetic is semantically correct
- Compliers can verify that there are no exceptions (errors) in the arithmetic of a computer program

Computer Hardware

Transreal arithmetic can make computer processors smaller

- There is no need to detect, signal, or handle arithmetical exceptions. This saves circuitry
- Even arithmetical underflow and overflow can be handled programmatically without hardware support
- Even memory handling can be handled programmatically without hardware support
- But I/O does need hardware support

Computer Hardware

The mathematical models on which digital computers are based are physically impossible

- Both the Turing Machine and the von Neumann Machine demand memory access in constant time, no matter the physical distance that has to be crossed for a processor to access memory
- Simulating this physically impossible model costs a great deal of circuitry, power, and time. In essence, conventional processors stall until memory is ready

Computer Hardware

- Data flow machines are physically possible
- Data flow machines using transreal arithmetic need never stall so they run much faster
- My team can design a computer chip with at least 4 000 transreal processing cores arranged as a data flow machine

Fun – Immovable and Irresistible

Suppose that an immovable object is subjected to an irresistible force. What happens?

- You could treat this as an exercise in physics and argue that there is no universe in which an immovable object and an irresistible force can co-exist
- You could treat this as an exercise in philosophy and debate wether the world is constrained to do only what is logically possible
- Or you could ask a 16 year old to set out the physical equations and ask a 12 year old to solve them using transreal arithmetic

Fun – Immovable and Irresistible

- An immovable object has infinite mass, $m = \infty$
- An irresistible force has infinite magnitude, $f = \pm\infty$
- The acceleration is $a = \dfrac{f}{m} = \dfrac{\pm\infty}{\infty} = \Phi$
- Thus, the immovable object accelerates at Φ units distance per unit time
- If the linking hypothesis is correct then the immovable object does not move and even an infinite force can be resisted

Fun – God

Suppose that God is omnipotent. Can God make a stone? Yes. Can God make a big stone? Yes. Can God make a stone so big that He cannot lift it?

- You could treat this as an exercise in physics and argue that God does not exist so the question is meaningless
- You could treat this as an exercise in philosophy and debate wether God is constrained to do only what is logically possible
- Or you could ask a 16 year old to set out the physical equations and ask a 12 year old to solve them using transreal arithmetic

Fun – God

- Let the biggest stone exert an infinite force, $f_s = \infty$
- Let God exert an infinite opposing force, $f_G = -\infty$
- The resultant force, $f_r = f_s + f_G = \infty - \infty = \Phi$
- If the linking hypothesis is correct then the stone does not experience a nett force and does not move
- In conclusion, God can make a stone so big that He cannot lift it and this is consistent with God giving the stone infinite gravity and resisting it with infinite muscular effort

Fun – Cosmology

Why is our universe of matter, energy, light and gravity finite?

- Suppose that $e = mc^2$ then there is a finite solution $e - mc^2 = 0$ in which all of the energy and matter in the universe can be created from zero energy and zero matter
- And there is a non-finite solution $e - mc^2 = \Phi$ in which all of the energy and matter in the universe can be created from nullity energy and nullity matter

Fun – Cosmology

- If $e = \Phi$ we may have $m, c = \Phi$, in which case the universe has not been created (from nullity)

- If $e = \Phi$ we may have $m = 0$ when $c = \pm\infty$ or $m = \pm\infty$ when $c = 0$, in which case matter or the speed of light has not been created (from zero)

- If $e = \pm\infty$ then $mc^2 = \pm\infty$ and at least one of m, c has infinite magnitude and the other has a non-zero and non-nullity magnitude, in which case an infinite universe has been created … but it is not going to last very long!

Fun – Cosmology

- A universe cannot have infinite opposing forces because these annihilate to nullity
- A universe cannot have infinite matter and gravity because any distribution of infinite matter, beyond a point, yields infinite opposing gravitational forces which annihilate
- A universe cannot have an infinite speed of light because photons accelerating matter move them to infinite speed in, generally, opposing directions and all of the infinite opposing forces in a collision annihilate

Fun – Cosmology

- Therefore, the only long-lived universe that can contain matter, energy, light and gravity is a finite universe

US Navy

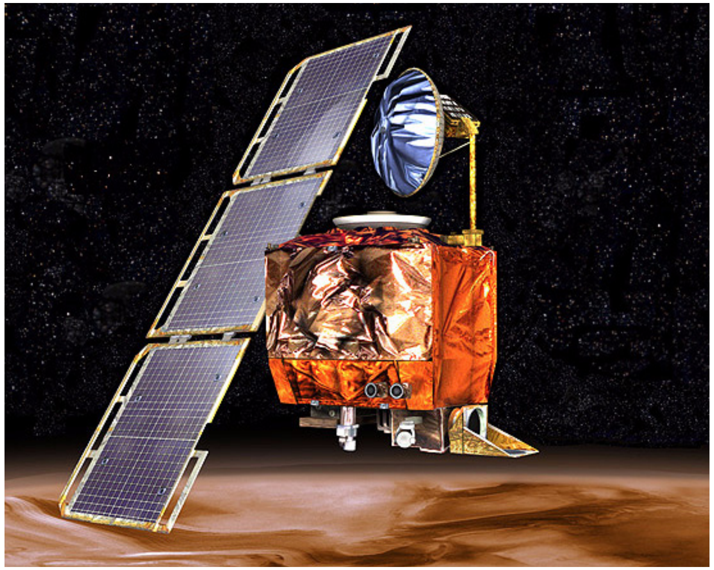
- The bridge of the missile cruiser, USS Yorktown, had networked computer control of navigation, engine monitoring, fuel control, machinery control, and damage control

US Navy

- On September 21st, 1997, a sailor on the USS Yorktown entered a zero into a database field, causing a division by zero error which cascaded through the ship's network, crashing every computer on the network, and leaving the ship dead in the water for 2 hours 45 minutes
- How much would the US Navy pay for computers that can divide arithmetically by zero?

Mars

- NASA's Climate Orbiter, which cost $125 M, crashed into the surface of Mars on 23 September, 1999, because a computer program mixed up foot-pound-second units with metre-kilogram-second units



- "People sometimes make errors," Edward Weiler, NASA's Associate Administrator for Space Science

Mars

- This bug could have been caught if the compiler had used dimensional analysis
- All ordinary type checking and ordinary dimensional analysis fail on division by zero
- But all syntactically correct sentences of transarithmetic are semantically correct – which means that a compiler can always check every possible evaluation of the transarithmetic in any program
- How much would NASA pay for a compiler that can always apply dimensional analysis?

Apocryphal Story

Apocryphal Story

- Once upon a time a Polaris missile was test fired. On launch it experienced severe turbulence and the guidance system instructed a maximal correction in the opposite direction to the turbulence
- The maximal correction happened to be the weird number. It was multiplied by -1, to be a correction in the opposite direction, but it stayed in the same direction, because it was the weird number
- Turbulence continued and the guidance system regained control of the missile's attitude and sent it on its ballistic course, on the *opposite* bearing from the one intended

Apocryphal Story

- How much would you pay to have strategic nuclear missiles fly in the right direction?

Paradigm Shift

The paradigm shift from real to transreal arithmetic has not yet sunk in to many parts of our society:

- The formal, computer proof systems *HOL Light* and *Mizar* make the assumption that $1/0 = 0$ in order to obtain total functions, but this is inconsistent with real (and transreal) arithmetic
- HOL light is widely used to prove the correctness of computer chips that are used in military and civil applications, including big science projects such as the Large Hadron Collider, the International Space Station, and all space vehicles
- These systems could use transreal arithmetic, but don't

Social Responsibility

- Transarithmetic extends mathematics, making it possible to solve problems that could not be solved before
- Transarithmetic challenges the use of real arithmetic in physics and the sciences
- Transarithmetic offers to make computer controlled machines safer

Social Responsibility

BUT

- When people move from real arithmetic to transreal arithmetic, things will get worse before they get better
- Moving too early will mean that there is no clear road ahead
- Moving too late will have put life and property at unnecessary risk and will have forfeited economic growth to competitors
- There are different Goldilocks points for different sectors of society. The Goldilocks point for computing is very, very early

Business Opportunity

If you invest £5 M in me and my team of professional chip designers, compiler writers, and managers then you are buying into a plan to:

- Fabricate and test a computer chip using transreal arithmetic within two years
- Build a supercomputer, using the chip, within the third year, that operates in the Peta FLOPS range and runs on a domestic power supply
- Roll out a series of products, with software products starting during the development phase